Solutions and Hints for the first three problem set

2.5

(a)

$$N \propto M_l^{-1.35} - M_u^{-1.35}$$

When $M_u >> M_l$, we have $N \propto M_l^{-1.35}$.

(b)

$$M_{total} = \int M\xi(M)dM \propto M_l^{-0.35}.$$

(c)

$$L_{total} = \int L(M)\xi(M)dM = \int M^\alpha \xi(M)dM \propto M_u^{\alpha-1.35} - M_l^{\alpha-1.35},$$

so as far as $alpha > 1.35$, the total luminosity is going to depend on bright stars, while the total mass on faint stars.

(d) when $M_l = 0.3$, the fracion for $M > 5$ is $(5/0.3)^{-1.35} = 0.02$, you can use combine solutions in (a) and (b) to get the total number of stars. Why do we see few stars in Figure 2.11 (open cluster) comparing to the number in 2.13? (globular cluster): light is dominated by the most massive star; for old systems, several things happened: (a) all AGB stars have similar luminosity regardless of mass; (b) IMF flattens in low mass end.

Stromgren sphere. (20 points) You will find that $R_s \sim 2AU$. What does that mean? One thing to consider is that the picture of Stromgren sphere is that the HII region has a sharp boundary: like a real sphere. So realistically, what's the thickness of HII region boundary? It has to be at least the mean free path of photons outside the sphere, the distance that it takes for a photon to travel in the neutral medium before it gets absorbed. The mean free path is defined as: $n_H \sigma l = 1$, where σ is the HI cross section $\sim 10^{-18}$ cm^2, so if $n_H = 1000$, then $l \sim 10^{15}$cm, which like about 100 AU. What that means is that photons can freely stream in the solar system, and it is always partially ionized, but doesn't have a sharp boundary such as in the case of a Stromgren sphere.

3.20

in order to have circular orbits, one needs $\partial\Phi_{eff}/\partial r = 0$. you can show that if $r < 3GM/c^2$, then $\Phi_{eff}/\partial r > 0$. Also to have solution for this quadratic equation, you need $L > 2\sqrt{3}GM/c$. To have stable orbits, one needs $\partial^2\Phi_{eff}/\partial r^2 > 0$.

4.5 what you need to do is to change all M in eq. (4.7) into rV_H^2/G, and note that for the first term in 4.7, r is (D-x). Then expand as in the textbook and assume $m << M$. See separate pdf for details.

4.6 1. mass within 20 kpc:

$$M = DV^2/G = 1.9 \times 10^{11} M_\odot$$

2. tidal radius:

$$r_J = D(m/(3M+m))^{1/3}$$

here r_J=5kpc, D=20kpc, and M is derived above. You will find, $m \sim 6 \times 10^9 M_\odot$.

3. mass to light ratio is (from Table 4.2 for luminosity) about 70.