Solutions and Hints for the fourth problem set

4.1. (30 points)

4.2. (30 points)

The easiest way is to start from definition (eq. 5.57)

$$\theta = d(1+z)/D$$

where

$$D = R_c sin(r/R_c)$$

for E-dS universe, $R_c = \infty$, so $D = r$, and r is the co-moving distance:

$$r = \int_{r_1}^{t_0} cdt/R(t)$$

Again, in an E-dS universe, $R(t) \sim t^{2/3}$, or $t \sim R^{3/2} \sim (1+z)^{-3/2}$. Plug those in, you can then show that:

$$r \sim 1 - (1+z)^{-1/2}$$

Then, find where $\partial\theta/\partial z = 0$.

4.3. (40 point)

This problem is asking you derive $\Omega(z)$ for a Λ universe. Many of you assume $\Lambda = 0$, rather than $\kappa = 0$. The correct way is:

$$\Omega(z) = \frac{8\pi\rho(z)}{3H(z)^2} = \Omega_0 \frac{(1+z)^3}{\Omega_0(1+z)^3 + \Omega_\Lambda}$$

this uses the fact that $\rho(z) = \rho_0(1+z)^3$, and $H(z)^2 = H_0^2[\Omega_0(1+z)^3 + \Omega_\kappa(1+z)^2 + \Omega_\Lambda$. and $\Omega_\kappa = 0$, and $\Omega_\Lambda = 1 - 0.3$. Then you will find that, at $\Omega(z = 0.5) = 0.59$, $\Omega(z = 10) = 0.998$,$\Omega(R = 10) = 4 \times 10^{-4}$.

5.1. (25 point)

Note that textbook seems to have an error. The correct answer should be $\Omega_r = 2\times10^{-5}h^2$.

5.2. (25 point)

5.3. (25 point)

5.4. (25 point) Integrate the Planck distribution, and you will find that $n_r \sim 20T^3\text{cm}^{-3}$.