Notes on Cosmological Parameters
AST 541, Fall 2014

We can rewrite Friedmann equation:

\[ \dot{R}^2 = \frac{8\pi G}{3} \rho R^2 - \kappa c^2 + \frac{1}{3} \Lambda R^2 \]

by dividing \( \dot{R}^2 \) in both sides, and ask: \( \Omega_{\Lambda} = \Lambda / 3H_0^2 \), and \( \Omega_k = -k c^2 / H_0^2 \), thus:

\[
\left( \frac{H(z)}{H_0} \right)^2 = \Omega_m (1 + z)^3 + \Omega_k (1 + z)^2 + \Omega_\Lambda.
\]

This is the form of Friedmann equation I find most useful, because it connects Hubble constant with other cosmological parameters: density, curvature and cosmological constant. Obviously, at \( z = 0 \):

\[ 1 = \Omega_m + \Omega_k + \Omega_\Lambda, \]

What Friedmann equation gives us is the expansion history of the universe. Through it, we also introduced a number of cosmological parameters.

- \( H_0 = \dot{R}/R|_0 \), is the current expansion rate of the universe.
- \( t_0 \) is the current age of the universe since the BB.
- \( \Omega_m \) is the current density parameter of the Universe.
- \( k \) is the curvature of the universe, deciding the geometry.
- \( \Lambda \) is the cosmological constant.
- \( q_0 \) is the deceleration parameter.

Clearly, they are not all independent. Indeed, they are tied by Friedmann equations. In matter dominated era, as the equation we showed above, the entire expansion history can be described by three parameters, including a scale (Hubble constant, or age of the universe), and two parameters that specify the relative contribution of matter (including dark matter), curvature and cosmological constant to the total energy-density budget of the universe. This is our Robertson-Walker-Friedmann world model. The most important task of a cosmologist is to understand what our world model is. And our biggest task is to test whether this world model, and which version of it, is supported by our observations. This is called the cosmological tests.
In fact, our cosmological test, or our cosmological model, includes even more parameters, because we are interested in not only the expansion history of the material dominated era, but (1) the state of the universe in radiation dominated era, i.e., CMB, (2) the growth of fluctuation in the universe.

This figure shows the number of parameters presented by the WMAP 7-year summary. The main conclusion of this is that our cosmology can be described by essentially 7 parameters that fit all existing observations satisfactorily. These observations are (almost all):

(1) the expansion history of the universe, including measurements such as supernova, which we will discuss next lecture;

(2) the anisotropy of the CMB;

(3) the large scale structure in low-z universe, including measurements of galaxy clustering and structure of the IGM, as well as clusters of galaxies

And all of them can be fit by seven parameters (paper said six, because it is assuming a flat geometry: $H_0 = 70$, $\Omega_m = 0.28$, $\Omega_b = 0.05$, $k = 0$, and then two parameters, the zero point and slope that specify the density fluctuation power spectrum, and one parameter describes the cross section of free electrons to CMB that will affect CMB photons through Compton scattering. We will discuss $\Omega_b$, and the other three later. For the moment, we will concentrate on the three parameters that appear in Friedmann equations, i.e., the expansion history of the universe, which we generally call the classical cosmological tests. But the cosmological tests will really dominate our discussions in this class in the next 1.5 months of the class. We will discuss how to measure $\Omega_b$ in the BBN lecture, and discuss CMB in details, as well as large scale structure tests in details later.

In the next two lectures, we will talk about: (1) Hubble constant; (2) age; and (3) $\Omega_m$ very briefly, and (4) $\Lambda$ and dark energy.

## 1 Hubble Constant: A History

The distance scale path has been a long and tortuous one, but with the imminent launch of HST there seems good reason to believe that the end is finally in sight. – Marc Aaronson

Hubble constant has sometimes been called the most variable constant in science. As an example of that, look at (Huchra Figure 1). A bit of history, the first published determination of Hubble constant is not from Hubble himself, but from Lemaitre in 1927 based on Hubble’s observations, and it is still the largest value to date, about 630 km/s/Mpc. Hubble himself
finally weighted in in 1929 at 500. Also, very early on, the Dutch astronomer, Jan Oort, thought something was wrong with Hubble’s scale and published a value of 290 km/s/Mpc, but this was largely forgotten.

The first major revision to Hubble’s value was made in the 1950’s due to the discovery of Population II stars by W. Baade. That was followed by other corrections for confusion, etc. that pretty much dropped the accepted value down to around 100 km/s/Mpc by the early 1960’s.

But after that, it entered an era. (Huchra Figure 2). So what is shown here is the Hubble constant determination from 1970 on. Note two symbols here, (1) open stars are values determined by two the de Vaucouleurs and his friend Sidney Van Ben Bergh. At least until 1990s, they always got H=100. (2) star symbols are values determined by another prominent group, by Allan Sandage, Hubble’s student and hand-picked heir, who might be the most important astronomer after Hubble, and his friend Gustav Tammam, a Swiss astronomer. For the past 40 years, their values were always 50 - 55.

Both can’t be right.

Then there are the third group of people, the people who initially didn’t not belong to these two camps. They are mostly “young” astronomers, at least when this group of astronomers entered the game in the 1970s. They invented a whole new arrays of distant indicators, and were much more adapted to the new technology at the time, such as CCDs. Their values more or less lied in the middle, like 75.

Then came HST. The single most important mission of HST is to measure Hubble constant, there is no doubt about that. The group that in charge of that is called the “Key Project” team. The measurement of Hubble constant is one of the two Key Projects of HST. Key project team used a combination of distance indicators, and did exclusively detailed work on the Cepheids in galaxies in Virgo clusters to calibrate their distance indicator. The final result of the HST Key Project, which was published about two years ago, was $72 \pm 8$. Note that recently Planck determined this value independently from CMB to be 69, in excellent reement. One of the most important goal of HST is to determine the Hubble constant within 10%, and they were able to do that.

$H_0$ is a difficult constant to determine. Let’s see why:

we have Hubble’s law:

$$cz = v = Hr$$

so the recession velocity, or the redshift, is proportional to the distance of a distant object.
In order to determine $H_0$, obviously, we need to determine two things: measure the redshift of a distant galaxy, and measure the distance to it with an independent method. So how far a galaxy can you use? We mentioned that the random motion of a galaxy due to its local gravitational field, i.e., the motion that has nothing to do with the Hubble flow, the uniform expansion of the universe, is of the order 500 - 1000 km/s. So in order to measure the Hubble constant that is not affected by this deviation from Hubble flow, we then have to go to sufficiently large distance when the velocity of the galaxy is dominated by the the expansion, not by the local velocity field. So in order to measure Hubble constant to an accuracy of 10%, we need to go to a expansion velocity of, say 7,000 km/s. If we assume that the Hubble constant is 70 km/s/Mpc, plug this back to Hubble’s law, we find that it required us to measure the distance of a galaxy at 100 Mpc away. This is very far away.

The other difficulty we have is that although using relations such as Tully-Fisher relation, Faber-Jackson relation and likes, we can measure the distance of faraway galaxies, these distance measurements are all Relative. It means that these relations more or less tell us how to determine the proportionality of galaxy properties. For example, T-F tells us that $L \propto v^4$, so if you measure the rotation curve of two galaxies, you can then know the relative intrinsic brightness of these two galaxies. Therefore, if you know the distance of one galaxy, and the relative observed brightnesses, then you can get the distance of the other galaxy. But it still requires you to know at least one galaxy, and therefore the relation needs to be calibrated. There are two flavors of distance indicators. Most of the distance indicators that we know of that apply to distant galaxies are relative distance indicators, they need to be tied to an absolute distance indicator in order to be applied to get Hubble constant. This process is very non-trivial. The simplest absolute distance indicator we know is the familiar trigonometric parallax. But it can only be applied to objects within few tens of parsecs with sufficient accuracy. On the other hand, in order to calibrate such as T-F relation, we need to measure the absolute distance to at least a few nearby spiral galaxies. We only know two spiral galaxies, M31 and M33, hardly enough. So we need to go to the nearest clusters of galaxies, such as Virgo, and Fornax clusters, which are a few Mpc away. To tie the distance of galaxies a few Mpc away with stars at few tens of parsecs away is very difficult.

The classical procedure for estimating distances in the Universe typically uses a series of relative distance indicators; the distance of a nearby galaxy might be determined by comparing the apparent brightness of its individual stars to those of similar stars in the Milky Way; the distance of those Milky Way stars are compared to some more nearby stars that we can use absolute distance indicators, and so on. This bootstrapping approach to measuring distances is called the cosmic distance ladder, where each rung up the ladder takes us to
greater distance, until we are out into the uniform Hubble flow to measure the distance indicated by the redshift.

You obviously DON’T want to have too many steps on the ladder. None of the distance indicator is perfect, they all have their errors, that is, the relation such as Cepheids, T-F, can not uniquely predict the distance, but only with a degree of uncertainty. When you begin to build the distance ladder, the error will build up, the relative error term more or less increases by the errors adding:

\[
(\sigma(D)/D)^2 = \Sigma(\sigma(D_i)/D_i)^2.
\]

So if you require the final Hubble constant to be 10% accurate, and you have 10 steps to reach it from trigonometric parallax in your distance ladder, than you need each step to be about 3% accurate, which is very hard to achieve.

Because the error budget is so tight, there are two things that people should worry about, (1) to find absolute distance indicator (other than parallax) to minimize the number of steps; (2) to try to make sure that the result is not biased, no systematic errors.

A few examples about measurements of large distances related to Hubble constant.

## 2 Absolute Distance Estimators

Absolute distance estimators uses simple geometric techniques, such as trigonometric parallax and moving cluster method. Now let’s see one more:

### 2.1 The Baade-Wesselink Method

If we know a star’s radius and its temperature, then it would be straightforward to measure its distance, since

\[
L = 4\pi r^2 \sigma T_{\text{eff}}^4,
\]

where \(\sigma\) is the Stephen-Boltzmann constant. However, to get radius is hard. This problem is possible to be overcome by applying it to variable stars. Lines in the spectrum of a variable star show Doppler shift, which tells us that the radius of the star changes, at the same time the luminosity changes. So :

\[
\Delta r \propto \int_{t_0}^{t_1} v(t) dt,
\]

And thus:

\[
\frac{L_1}{L_0} = \frac{(r_0 + \Delta r)^2 T_1^4}{(r_0)^2 T_0^4}.
\]
This is the basics of the Baade-Wesselink method. It seems straightforward, although its application is not, because the pulsation of a star is a complicated process, and if you remember in your star class, the effective radius of a spectrum line, and of the continuum, which we use to measure the temperature is different, the expansion of a pulsating star might not be uniform etc. So applying this method needs very careful modeling of the star in question.

A more important application of this technique is to use it on calibrating the supernova distance, which is now becoming one of the most important distance indicators, because you can observe it at cosmological distant, up to $z = 1.8$ now.

One golden opportunity is SNe research is the supernova 1987A. I don’t know how many of you are old enough to remember it. I was in high school at the time. 1987A occurred in LMC, and it is the first visible SN in more than 300 years, so it is the first SN to get proper modern observation. And because it is in the LMC, a place where a lot of extragalactic distance indicators got calibrated, an accurate distance to LMC is in particular important. Using B-W method on 1987A needs careful modeling. People got the distance to LMC to be $55 \pm 5$ kpc in this case.

The advantage of this method is obvious, it can be applied directly to high redshift, to cosmological distant, there is no assumption about the radiation mechanism, such as in the case of B-W method, or about symmetry (perfect ring) such as in the SN case. The physics is simple and clean, sounds very good, right?

The real situation is more complicated, as always. Here the difficulty is partly observational. It turns out that it is not easy to measure the time delay; quasar varies very irregularly, and the time delay is typically hundreds of days; optical monitor of quasars is always affected by weather and availability of the object on the night sky. So there had been a series of controversy about $H_0$ measured this way, and the value used to be on the small side, although it is catching up lately.

Another example of absolute distance estimators is the Sunyaev-Zeldovich effect. The decrement of CMB due to the Compton scattering is proportionally to the linear size of the cluster, and the angular size of the cluster can be measured by X-ray imaging, thus the distance.

These are some of the primary ways to measure distances absolutely. The advantage is that there is no intermediate steps involved, so every one of these methods are independent. And a number of them, lensing, and S-Z, can be applied directly to cosmological distances without intermediate step, which is great. The disadvantage of almost all of these method, esp. those that can be applied to large distance, is that they required a fair amount of modeling, and
assume things like certain perfect geometry, and symmetry, examples such as we need assume a perfectly rotating disk in the case of maser, a spherical cluster for S-Z, and certain mass model for lensing galaxy in time delayed lensed quasars. So while the physics is clean, and for a number of them, the accuracy is high, they are all prone to systematically errors, errors that occurred in your assumption of the physical setups. This has always been the problem with using them to determine $H_0$ directly from the first principle. And a lot of people favor a more conservative approach, by carefully calibrating a series of distance indicators and building the distance ladder along the way. In order to do that, we need to understand more about these relative distance indicators.

3 Relative Distance Estimators

We probably know a good number of them. Such as Cepheid variables, Tully-Fisher relation, Fundamental plane relation, Surface brightness fluctuations etc.

3.1 Variable Stars

in particular pulsating variables, Cepheids for Population I, and RR Lyrae stars for Pop II. Stellar structure theory predicts relation of the variable period and the luminosity of the pulsating variables. Since variation period is independent of distance, one can get luminosity independently.

To establish Cepheids variable distance to a large number of nearby galaxy is the chief goal of the HST $H_0$ key project. Cepheids are very bright, $M_v \sim -3$, and can be measured to $\sim 10 \, \text{Mpc}$ scale using HST. The major problem of using Cepheids has been to calibrate the metallicity dependence of the period-luminosity relation, that is, high metallicity Cepheids are somewhat brighter than low metallicity ones, which will introduce systematic errors.

Cepheids (and RR Lyraes) are relative distance estimators. The stellar structure theory doesn’t provide us an absolute calibration of their distances. In fact, their distances are calibrated with a number of cepheids in galactic clusters, whose distances are calibrated using main sequence fitting technique against very nearby clusters such as Pleiades, whose distance can be measured with parallax or moving cluster methods.

A common feature of all relative distance estimators is that they are all based on some more or less empirical relations from observations. Some of them have a very clean physical explanation, some remain to be more or less empirical.
4 Results

The HST key project, whose goal is to measure $H_0$ with 10% accuracy, was formed in 1986, by Marc Aaronson, and started their observations in 1991. The final result of the key project is published in 2001. The key project way to determine $H_0$ is (1) to find and calibrate Cepheid distance in a large number of nearby (< 20 Mpc) galaxies, and use it as the primary distance indicator. and (2) then to use T-F, F-P, type Ia, and SBF as the secondary indicator to greater distance to determine $H_0$. They were able to achieve both goals. With the Cepheids calibration, (Table 1), they were able to much better calibrate the secondary indicators, the error on these indicates decreased from 10-20%, to 4-10%, because of that.

This plot (Figure 4) shows Hubble diagram, the diagram that shows the relation between distance and velocity, of key project results. Hubble diagram is a very important concept, which you should all remember. It plots D vs. v, and the slope of Hubble diagram is $H_0$. Also, when you go really far, it is going to curve, deviate from a straight line due to GR effect.

So this figure shows the Hubble diagram, with different indicators of Hubble constant, T-F, F-P, SBF, Ia and II (expanding photosphere). They are all consistent, no systematic differences among them. Table 12 shows the values and uncertainties. The final answer from the Key Project team is.

$$H_0 = 72 \pm 8 \text{km s}^{-1}\text{Mpc}^{-1}.$$  

5 Age of the Universe

Deceleration makes the universe younger than a Hubble time. Looking back in time, the expansion was faster and therefore we reach the Big Bang more quickly. Draw pictures.

For $\Omega = 1$ universe, the age of the universe is $2/3H_0$. Alternatively, $H_0t_0 = 2/3$.

For open universes without lambda, $H_0t_0$ can be larger. An empty universe $\Omega_m = 0$ has $H_0t_0 = 1$; this is sometimes called a coasting universe. For reasonable $\Omega_m = 0.3$ or so, the result is about 0.85.

Positive $\Lambda$ allows $H_0t_0 > 1$. In fact, as $\Omega_m \rightarrow 0$, the Hubble parameter becomes time-independent, $\dot{R} = H_0R$ means that $R \propto \exp(H_0t)$, and we have an infinitely old universe. More on that later!

Lesson: Lower matter densities yield less deceleration and hence older universes for a fixed Hubble constant.
Observationally, the prejudice toward $\Omega = 1$ caused higher Hubble constants to be in conflict with age estimates of old stellar populations, in particular globular clusters. $H_0 = 50$ km/s/Mpc and $\Omega = 1$ gave an age of 13.3 Gyr. But $H_0 = 70$ would be only 10 Gyr. This was called the ‘age crisis’.

Globular cluster ages: typically people say that the ages are 15 Gyr. Like many branches of astrophysics, errors bars have been quoted that in time turn out to have neglected important systematic effects. Opacities, abundance ratios, and the local distance scale have all caused these estimates to move around in the last 10 years.

One can also try to infer ages from white dwarf cooling.

With the currently favored $\Lambda$ universes, the age of the universe is about 13–14 Gyr. My outsider’s opinion is that this is within the errors on the stellar ages. I would also stress that it’s remarkable that two completely disjoint programs agree to about 10%.

6 $\Omega_m$

The debate on the value of $\Omega_m$, the mass density, is a quite old one. The first attempt to measure $\Omega_m$ is to measure the deceleration of the universe to high redshift, using something called the brightest cluster galaxies, the brightest member of a cluster, which seems to be a quite good (10-15%) relative distance indicator. So what they were doing is really to measure $q_0 = 1/2\Omega_m$ if $\Lambda = 0$. The game started from as early as late 1950s, but it was never successful. The resulted $q_0$ tends to be all over the place, and it turned out that at that time, people didn’t know how to model the evolution of galaxy luminosity itself, and at high redshift, the properties of BCGs changed, they are younger and brighter, so they are not good standard candles. In the end, this test didn’t go anywhere. But such attempts unexpectedly grew into a whole new field, the field of galaxy evolution, when a group of young astronomers in mid 1970s began to question this method.

The second setback for measuring $\Omega_m$ is the realization that the universe is dark-matter dominated, and the visible, baryonic matter is but only a small fraction of it. So by counting stars and galaxies, it is very difficult then, to add up the total mass density of the universe, and get $\Omega_m$ in this way.

Because of the difficulty of measuring $\Omega_m$ with deceleration, and with counting galaxies, for a long time, it is a rather confusing field. On one hand, people have known for a long time that baryonic matter is only a few percent, $\Omega_b < 0.05$. On the other hand, the nature of dark matter prevented people how to correctly estimate total $\Omega$ for a while. By 1980s, there
are two obvious camps, one favors $\Omega = 1$, on the basis that it is the most elegant universe, inflation predicted it, flatness problem solved and so forth, the other favors $\Omega < 0.5$, as they just can’t come up with enough dark matter in their measurement to close the universe. Actually, the majority astronomers who thought they knew the answer believed $\Omega_m = 1$. But in this case the minority won.

In the 1990s, the possibility of a low-mass universe gained substantial support through a number of very convincing observations, many of them independent. Although each observation has its strengths, weaknesses and assumptions, they all indicate that $\Omega_m < 1$.

Most of the methods that directly measure $\Omega_m$ are based on observations of clusters of galaxies. We will defer those to much latte because they require us to understand how cluster of galaxies are formed.

So by late 1990s, the overwhelming evidence is that $\Omega_m$ is small, we live in a light weighted, low density universe; but as I explained last time, theorists really want the universe to be flat, and with our cosmic triangle,

$$1 = \Omega_m + \Omega_\kappa + \Omega_\Lambda,$$

a low density universe can not be flat unless there is a cosmological constant. Is there? Now let’s turn our attention to the second side of the cosmic triangle, the cosmological constant term, which determines the acceleration and the fate of the universe.

7 Measuring $\Lambda$, the accelerating universe

Nobel Prize!

The discovery, of course, comes from detailed observations of type Ia supernovae, which we said in last lecture, that are probably the best distance estimator there is for very distant object. Let me remind you, that type Ia SNe are SNe without H or He, they are thought to be accreting WDs that goes over Chandra limit and blow up. Because the progenitors are so similar, they are thought to have very similar total luminosity. And since the peak luminosity of a SN can be much higher than that of the whole galaxy, it can be observe to cosmological distance. The simple stellar physics argues that the estimator is not likely to evolve with time, that is, it is a good estimator now as it is far away, we don’t need to worry about evolution.

One more trick (Figure) will make it an even matter estimator. This figure shows a collection of SN lightcurves for nearby objects. You can see that do not all have the same peak absolute
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magnitude, some are brighter than others. You will also notice that the fainter ones tend to have much steeper light curves than the brighter ones. So by using the steepness of the light curve as a correction factor, you can greatly improve this distant estimate, as shown on the bottom half of the figure. After you make this adjustment, it is a much more uniform estimator now!

So the idea is to use them as standard candle, and find them at high redshift, plot them on Hubble diagram, which I remind you are the diagram that shows the relation between velocity, or redshift, and distance, or distance modulus. So this figure (old figure) shows the relation between the distance modulus (of the luminosity distance) and redshift, and models with different $\Omega_m$ and different $\Lambda$ will follow different curves. Then by looking at them at high redshift, you can constrain the combination of $\Omega_m$ and $\Lambda$. As I said before, the slope of the Hubble diagram tells you the expansion rate of the universe, or Hubble constant; then the curvature of this relation, tells you the second order effect, or the acceleration of the universe, $\ddot{R}$. So distant SNe are primarily used to measure the acceleration of the universe, which is a combination effect of $\Omega_m$ and $\Lambda$.

To find them, especially a lot of them, at cosmological distant, however, is not easy. First, they are faint, second, they are rare, a typical galaxy has only one per several hundred years. Third, they appear as random, with no warning where to look.

* But the scarce observing time at the world’s largest telescopes, the only tools powerful enough to measure these most distant supernovae adequately, is allocated on the basis of research proposals written more than six months in advance. Even the few successful proposals are granted only a few nights per semester. The possible occurrence of a chance supernova doesn’t make for a compelling proposal.

* They are fleeting. After exploding, they must be discovered promptly and measured multiple times within a few weeks, or they will already have passed the peak brightness that is essential for calibration. It’s too late to submit the observing proposal after you’ve discovered the supernova.

This was a classic catch-22. You couldn’t preschedule telescope time to identify a supernova’s type or follow it up if you couldn’t guarantee one. But you couldn’t prove a technique for guaranteeing type Ia supernova discoveries without prescheduling telescope time to identify them spectroscopically.

In fact, the results from the first search for very distant type Ia supernovae were not encouraging. In the late 1980s, a Danish team led by Hans Nielsen found only one type Ia supernova in two years of intensive observing, and that one was already several weeks past its peak.
The solution for this dilemma is to form a large team, convince a large number of your colleagues, who have access to an array of telescopes, to work together. And also convince your observatory director to be able to intervene the regular program when the opportunity comes. For the initial discovery of these SNe, typically, only a small telescope is needed, and you can get a lot of time to monitor a large number of galaxies, to find possible SNe. Then go to a larger telescope to take spectrum, to determine whether or not it is a type Ia, and what is the redshift; this needs things like Keck. Finally, when the object is fading away, or for the most distance ones, you will need Hubble to do that. So it is a big operation and a complicated campaign.

There are two teams in the world that does that. One is called the Supernovae Cosmology Team, based in Lawrence Berkley Lab, also include astronomers from all over the world, the other called the High-z SN team, consists of people from Harvard, Berkely, HST, Australia etc. Now let’s see a few plots from them: Figure: Hubble diagram from SCP, from Science. If the universe is empty, Newton’s first law, it is going to have a constant expansion, no change in $\dot{R}$. For a matter dominated universe, it will decelerate. However, this figure shows that the data are consistent with the universe accelerating, and the only possibility, as we showed before, is there is a component acting like a repulsive force, i.e. cosmological constant.

Figure, hz_highzhub_col_bothbig.gif

Figure, constrain on parameter. It is clear that the acceleration of the universe depends on the relative strength of matter - decelerating, and cosmological constant - accelerating. So by measuring the distance of SNe, and plotting Hubble diagram like this, we will obtain a combined constraint on $\Omega_m$ and $\Lambda$.

From these observations, it shows very strong evidence that the universe is accelerating, that there is a positive cosmological constant. But could it be due to some systematic error made in their observations?

### 7.1 Systematics??

Crucial questions of replicability were answered by the striking agreement between our results and those of the competing team, but there remain the all-important questions of systematic uncertainties. Most of the two groups’ efforts have been devoted to hunting down these systematics. Could the faintness of the supernovae be due to intervening dust? The color measurements that would show color-dependent dimming for most types of dust indicate that dust is not a major factor. Might the type Ia supernovae have been intrinsically fainter
in the distant past? Spectral comparisons have, thus far, revealed no distinction between the exploding atmospheres of nearby and more distant supernovae.

Another test of systematics is to look for even more distant supernovae, from the time when the universe was so much more dense that \( \rho \) dominated over the \( \Lambda \) and was thus still slowing the cosmic expansion. So although the universe is accelerating now, at some point at high redshift, it must have been decelerating. Supernovae from that decelerating epoch should not get as faint with increasing distance as they would if dust or intrinsic evolutionary changes caused the dimming. The first few supernovae studied at redshifts beyond \( z = 1 \) have already begun to constrain these systematic uncertainties.

Figure: Reiss plot.

This is the most distant type 1a SN yet discovered. It is found in the famous Hubble Deep Field, quite by accident a few years after the data was taken. What it shows is that even though the error bar is still very large, this most distant SN does show it is brighter than a constantly expanding universe is, and the universe is decelerating then. Since it is very difficult to have any systematic effect such as dust, or intrinsic evolution, to affect the measurement at \( z \sim 0.5 \) in one way, and \( z \sim 1.5 \) in the other way, it might be the strongest evidence yet on the existence of \( \Lambda \).

The third side of the cosmic triangle is its flatness, mostly constrained by the CMB measurement. We will postpone this discussion to after the CMB lecture.

8 Dark Energy

What’s the meaning of \( \Lambda \)? It is a repulsive force in the universe that accelerating the universe. Looking at the energy equation of the Friedmann equation:

\[
\ddot{R} = -\frac{4\pi G}{3} R \left( \rho + \frac{3p}{c^2} \right) + \frac{1}{3} \Lambda R
\]

First, it causes the universe to accelerate. Second, it looks like some sort of energy term just as matter or photons... but there is nothing there. In elementary particle physics, it represents some sort of vacuum energy.

The story might stop right here with a happy ending– a complete physics model of the cosmic expansion– were it not for a chorus of complaints from the particle theorists. The standard model of particle physics has no natural place for a vacuum energy density of the modest magnitude required by the astrophysical data. The simplest estimates would predict a
vacuum energy 10120 times greater.

(In supersymmetric models, it’s “only” 1055 times greater.) So enormous a $\Lambda$ would have engendered an acceleration so rapid that stars and galaxies could never have formed. Therefore it has long been assumed that there must be some underlying symmetry that precisely cancels the vacuum energy. Now, however, the supernova data appear to require that such a cancellation would have to leave a remainder of about one part in 10120. That degree of fine tuning is most unappealing.

The cosmological constant model requires yet another fine tuning. In the cosmic expansion, mass density becomes ever more dilute. Since the end of inflation, it has fallen by very many orders of magnitude. But the vacuum energy density $rL$, a property of empty space itself, stays constant. It seems a remarkable and implausible coincidence that the mass density, just in the present epoch, is within a factor of 2 of the vacuum energy density.

Given these two fine-tuning coincidences, it seems likely that the standard model is missing some fundamental physics. Perhaps we need some new kind of accelerating energy— a ”dark energy” that, unlike $\Lambda$, is not constant. with the goal of solving the coincidence problems.

The dark energy evinced by the accelerating cosmic expansion grants us almost no clues to its identity. Its tiny density and its feeble interactions presumably preclude identification in the laboratory. By construction, of course, it does affect the expansion rate of the universe, and different dark-energy models imply different expansion rates in different epochs. So we must hunt for the fingerprints of dark energy in the fine details of the history of cosmic expansion.

The wide-ranging theories of dark energy are often characterized by their equation-of-state parameter

$$w = \frac{p}{\rho c^2},$$

the ratio of the dark energy’s pressure to its energy density. The deceleration (or acceleration) of an expanding universe, given by the general relativistic energy equation:

$$\ddot{R} = -\frac{4\pi G}{3} R (\rho + 3p/c^2) + 1/3 \Lambda R.$$

In case we don’t have cosmological constant:

$$\ddot{R} = -\frac{4\pi G}{3} R (\rho + 3p/c^2) = -\frac{4\pi G}{3} \rho (1 + 3w).$$

So the expansion of the universe would depend on this ratio $w$. Thus the expansion accelerates whenever $w$ is more negative than $-1/3$, after one includes all matter, radiation, and dark-energy components of the cosmic energy budget.
Each of the components has its own $w$: negligible for nonrelativistic matter, obviously, for them, $p = 0$, pressureless. $w = +1/3$ for radiation and relativistic matter, for them $p = \rho/3c^2$. and $w = -1$ for $\Lambda$. Let’s say how this is the case:

$$\Omega_{\Lambda} = \Lambda/3H^2,$$

We can write:

$$\Omega_{\Lambda} = 8\pi G \rho_{\Lambda}/3H^2,$$

similar to the way we write $\Omega_m = 8\pi G \rho/3H^2$. So $\Lambda = 8\pi G \rho_{\Lambda}$. Now for the energy equation, if we have only $\Lambda$ term:

$$\ddot{R} = 1/3\Lambda R = 8/3\pi R \rho_{\Lambda},$$

so comparing this with the equation above, we then have:

$$-2 = 1 + 3w \rightarrow w = -1.$$ 

That is, $\Lambda$ exerts a peculiar negative pressure! General relativity also tells us that each component’s energy density falls like $R^{-3(1+w)}$ as the cosmos expands. Therefore, radiation’s contribution falls away first, so that nonrelativistic matter and dark energy now predominate. Given that the dark-energy density is now about twice the mass density, the only constraint on dark-energy models is that $w$ must, at present, be more negative than $-1/2$ to make the cosmic expansion accelerate. However, most dark-energy alternatives to a cosmological constant have a $w$ that changes over time. If we can learn more about the history of cosmic expansion, we can hope to discriminate among theories of dark energy by better determining $w$ and its time dependence. Unfortunately, the differences between the expansion histories predicted by the current crop of dark-energy models are extremely small. Distinguishing among them will require measurements an order of magnitude more accurate than those shown in figure 3, and extending twice as far back in time. There are many telescope, both ground-based, and from space, that plan to observe many more distance SNe, to constrain the expansion of the universe much better, and to constrain the equation of state.

9 Cosmic Dynamics Experiment

We will discuss other dark energy experiments later on in this class. But I do want to mention one of the more ambitious ideas that I encountered. Issue with many DE experiments is that they suffer from systematics. SN tests you always worry about dust and evolution; other tests based on large scale structure have subtle effect with the growth of structures and the bias of tracers. People always wonder about what if one can directly measure the
acceleration of the universe. Here direct means measure the second derivative of scale factor, or first derivative of redshift, or, basically wait long enough to see the redshift of the object change with time. You can show (which is a homework problem) that:

\[ \frac{dz}{dt} = (1 + z)H_0 - H(z), \]

so for an object at rest, if you wait long enough, then because of the acceleration of universe, the line will shift. If you observe this over a range of redshift, then you are mapping \( H(z) \), or Friedmann equation directly. The only problem is that this shift happens at Hubble time scale, i.e., \( \frac{dz}{dt} \) is extremely small. \( \sim 10^{-10} \) per year, requiring velocity accuracy of cm/s, or about two orders of magnitude better than exoplanet surveys have achieved.

But that didn’t stop people from planning. In fact, this is one of the science drivers for E-ELT.... Use Ly\( \alpha \) forest observations. Very stable. 4000 hours on E-ELT over 20 years for one set of measurements. 10% of time available. Accuracy and model independent measurements of expansion history. Worth it?