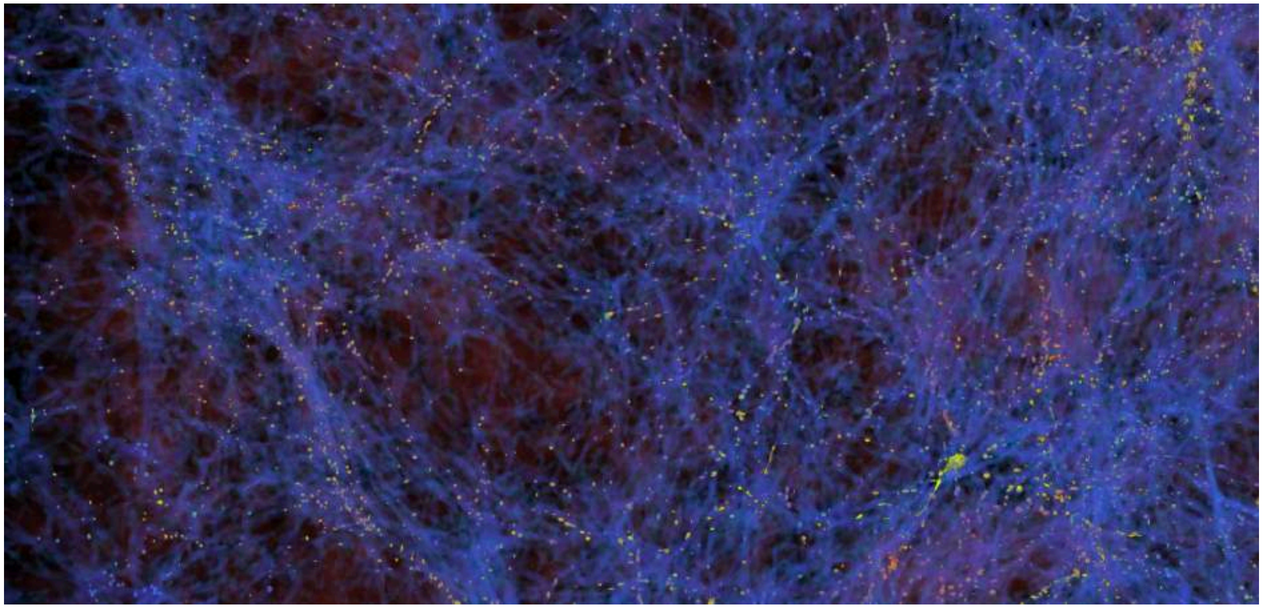
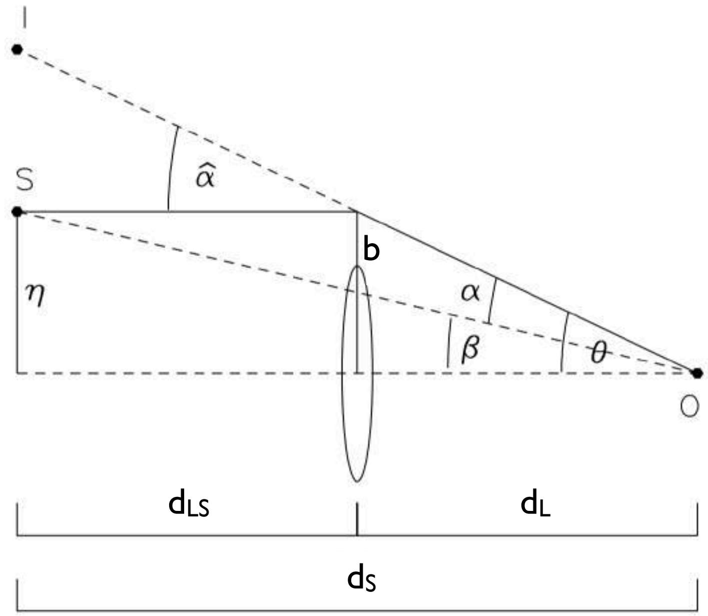
Part 2: Lensing by distributed masses

- Theory
  - Cluster mass
  - Subhalos

- Applications:
  - Search for subhalos
  - Transients: SN Refsdel and Icarus
  - Cosmography



Part 2: Lensing by distributed masses - Cluster



- Consider an extended, axially-symmetric lens. Deflections are radial & the mass *M(<b)* matters:

$$\hat{\alpha} = \frac{4GM(<b)}{c^2 b}$$

- Recall the lens equation: $\theta - \beta = \frac{d_{LS}}{d_S}\hat{\alpha}$. The deflection is no longer simple. On substituting:

$$\theta - \beta = \frac{d_{LS}}{d_S}\frac{4GM}{c^2 b}$$

- Let us say this lens has constant surface mass density, Σ, such that $M = \Sigma\pi b^2$. On making this substitution, recalling that $b = d_L\theta$, and solving for β:

$$\beta = \theta\Big(1 - \frac{4\pi G\Sigma}{c^2}\frac{d_L d_{LS}}{d_S}\Big)$$

- Now, β ~ θ. If we set $\Sigma_{cr} = \frac{c^2}{4\pi G}\frac{d_S d_L}{d_{LS}}$, then β = 0 for all θ. When $\Sigma = \Sigma_{cr}$, we get giant arcs

Part 2: Lensing by distributed masses - Cluster

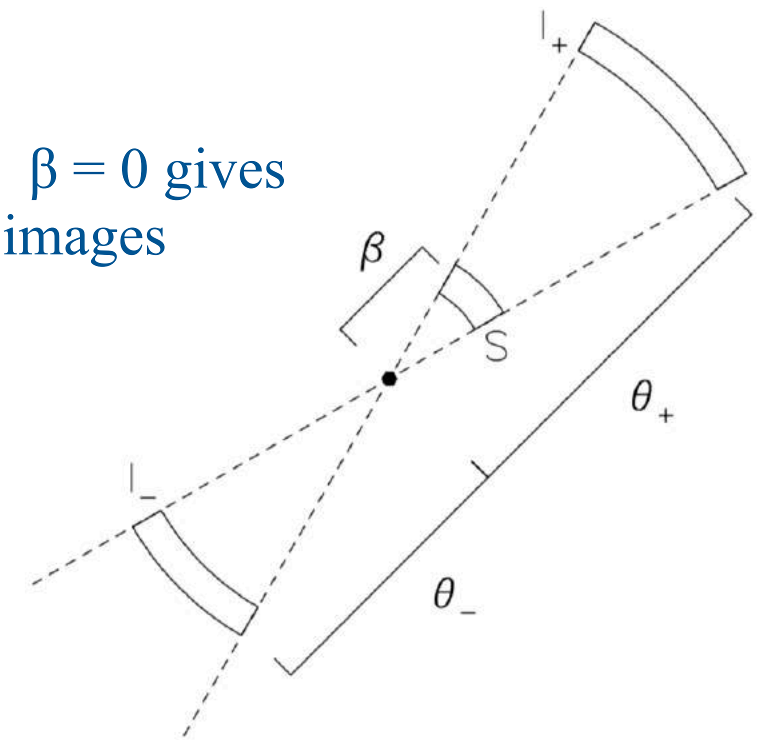
- For the canonical singular isothermal sphere:

$\rho(r) = \frac{v_c^2}{4\pi G r^2}$, which has projected surface mass density: $\Sigma(r) = \frac{v_c^2}{4Gr}$

- Then the mass interior is: $M = \int_0^R \Sigma(r) 2\pi r dr = v_c^2 \pi R/2G$

- So the deflection angle is: $\hat{\alpha} = \frac{4GM}{c^2 R} = \frac{2\pi v_c^2}{c^2}$ <== independent of radius!

- The lens equation is: $\theta - \beta = \frac{d_{LS}}{d_S} \frac{2\pi v_c^2}{c^2} = \pm\theta_E$. $\beta = 0$ gives the Einstein ring. Otherwise, again we get +/- images

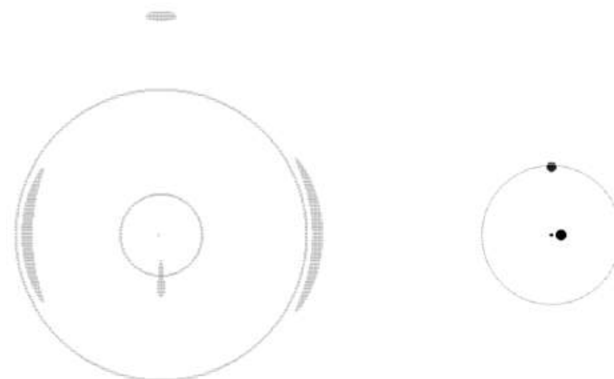


- There is an I_+ at $\beta > \theta_E$ and I_- at $\beta < \theta_E$. I_+ is brighter than I_-, and I_- is parity flipped. Note for $\theta > 2\theta_E$ there is only one image

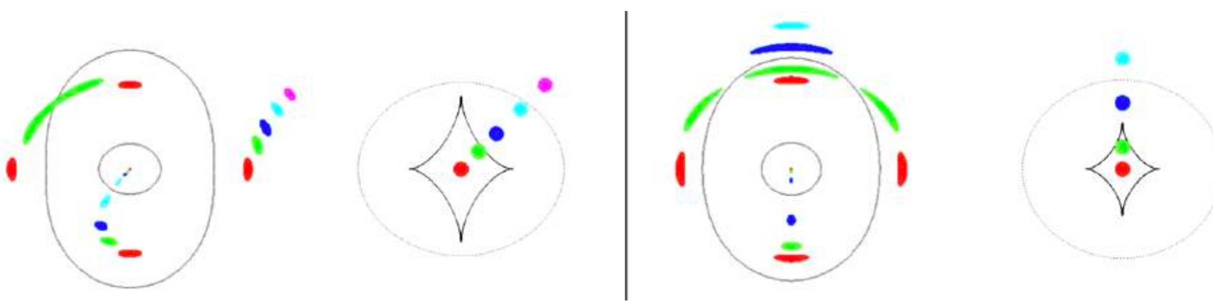
Part 2: Lensing by distributed masses - Cluster

- Here is an example of the imaging of an extended source by a non-singular circularly-symmetric lens.

- A source near to the center produces two tangentially-stretched images, and a source on the caustic produces one radial arc and one tangential arc.



- real mass distributions are not circularly-symmetric. For this elliptical lens, there can be 4 or even more images. Here source plane has regions where odd-numbered images are created, yet one image is demagnified



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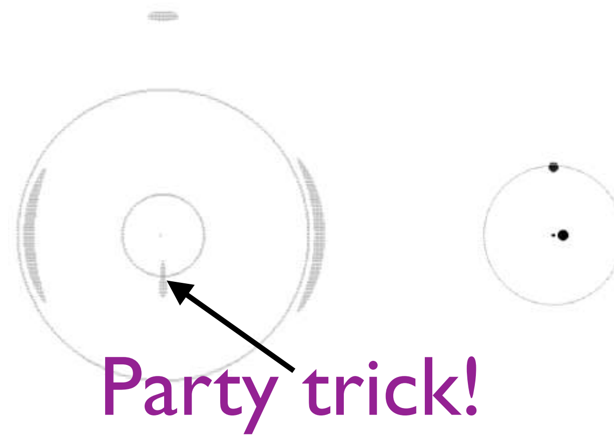

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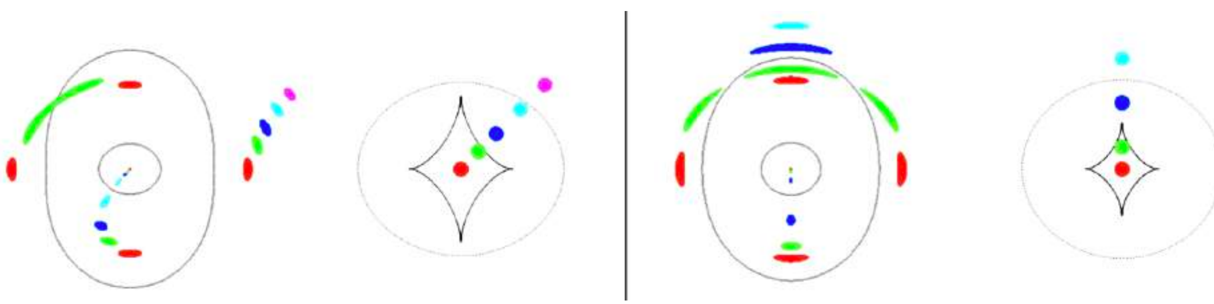
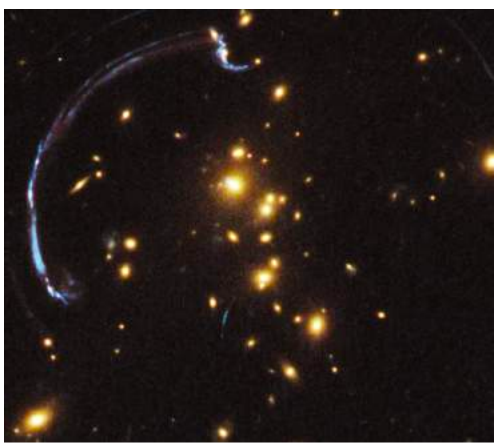
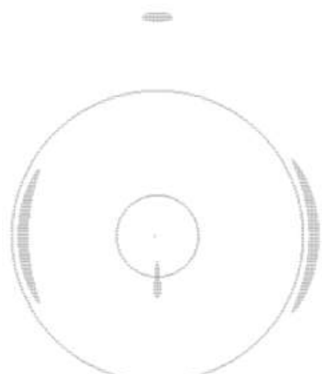


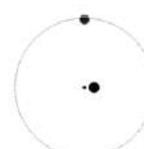
Part 2: Lensing by distributed masses - Cluster

- Arc locations give projected cluster mass inside circle traced by the arc
- For a circularly-symmetric lens, Σ is approximately equal to Σ_{cr}
- The radius gives an estimate of Einstein radius of the cluster:

$$M(\theta) = \Sigma_{cr}\pi(d_L\theta)^2 \approx 1.1 \times 10^{14} M_\odot \left(\frac{\theta}{30''}\right)^2 \left(\frac{D}{1Gpc}\right)$$







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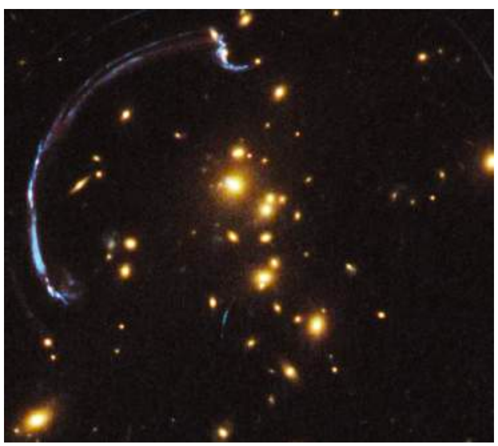
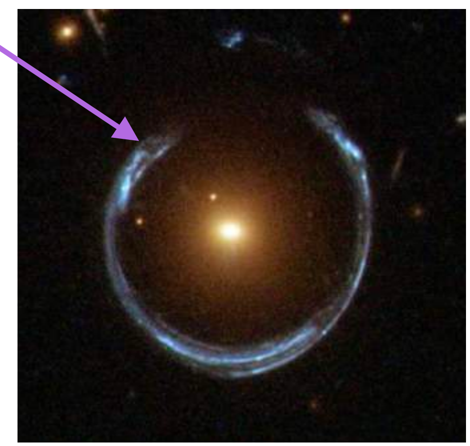
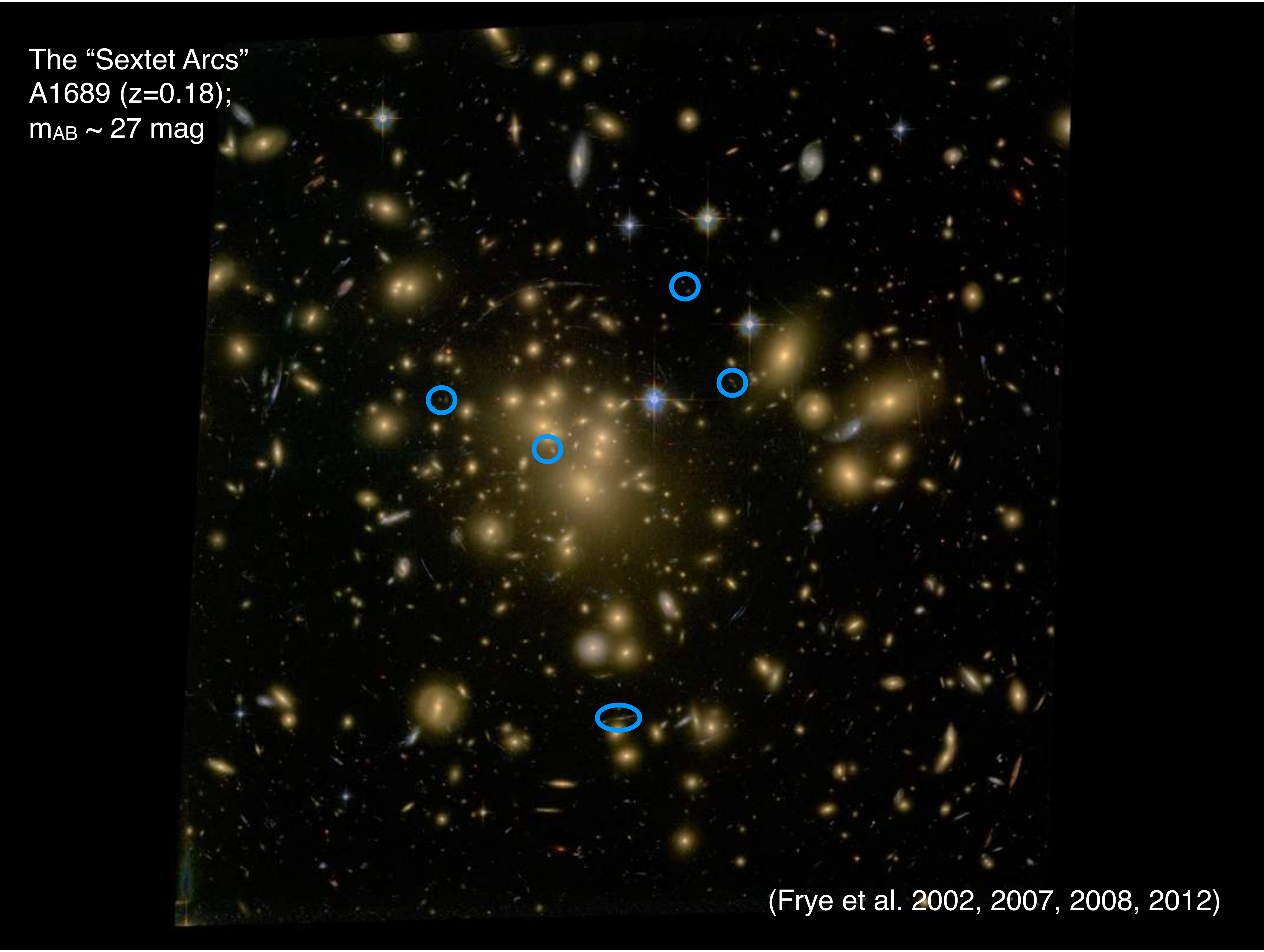


Party trick!




The “Sextet Arcs”
A1689 (z=0.18);
$m_{AB} \sim 27$ mag
(Frye et al. 2002, 2007, 2008, 2012)

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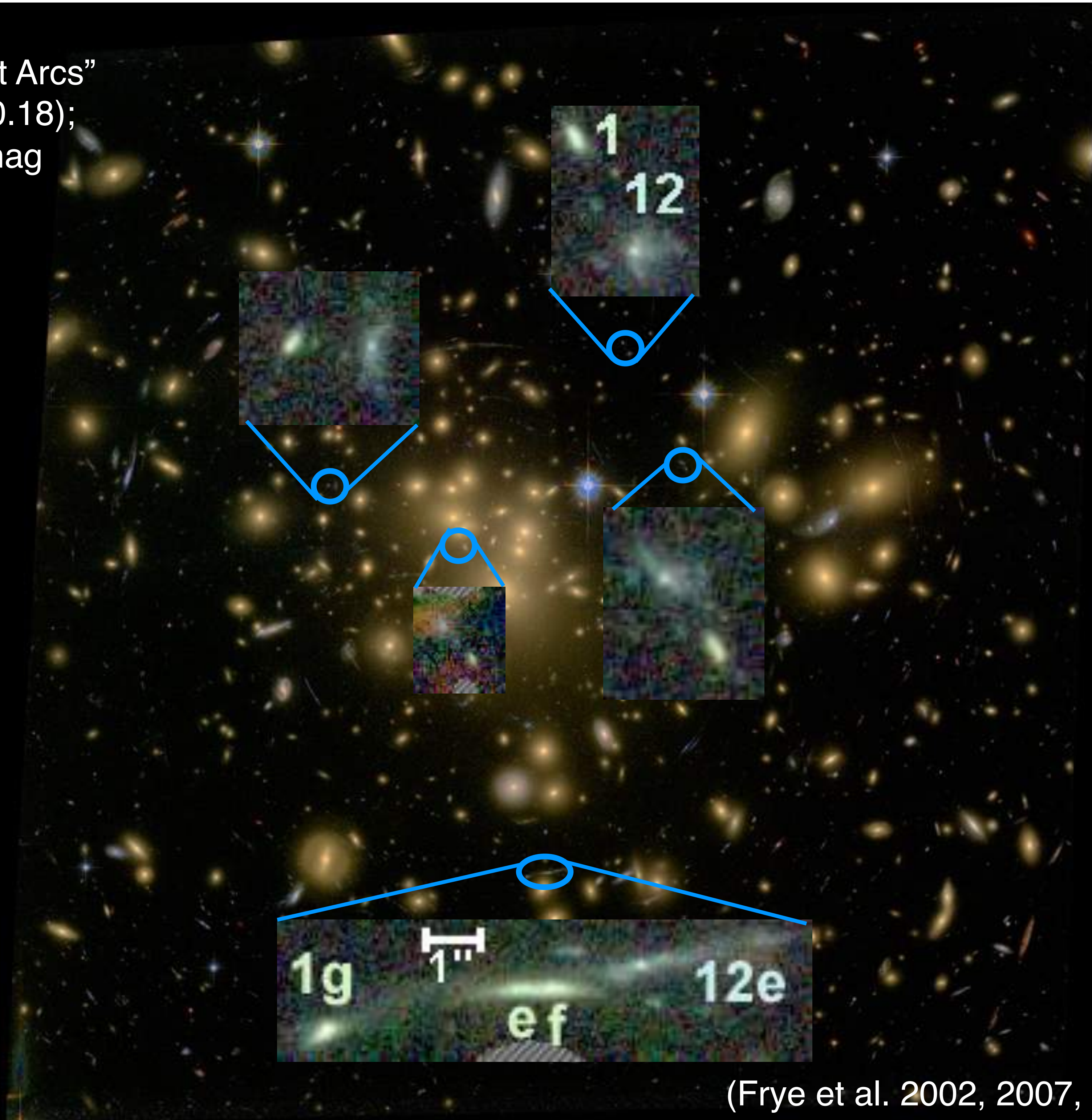
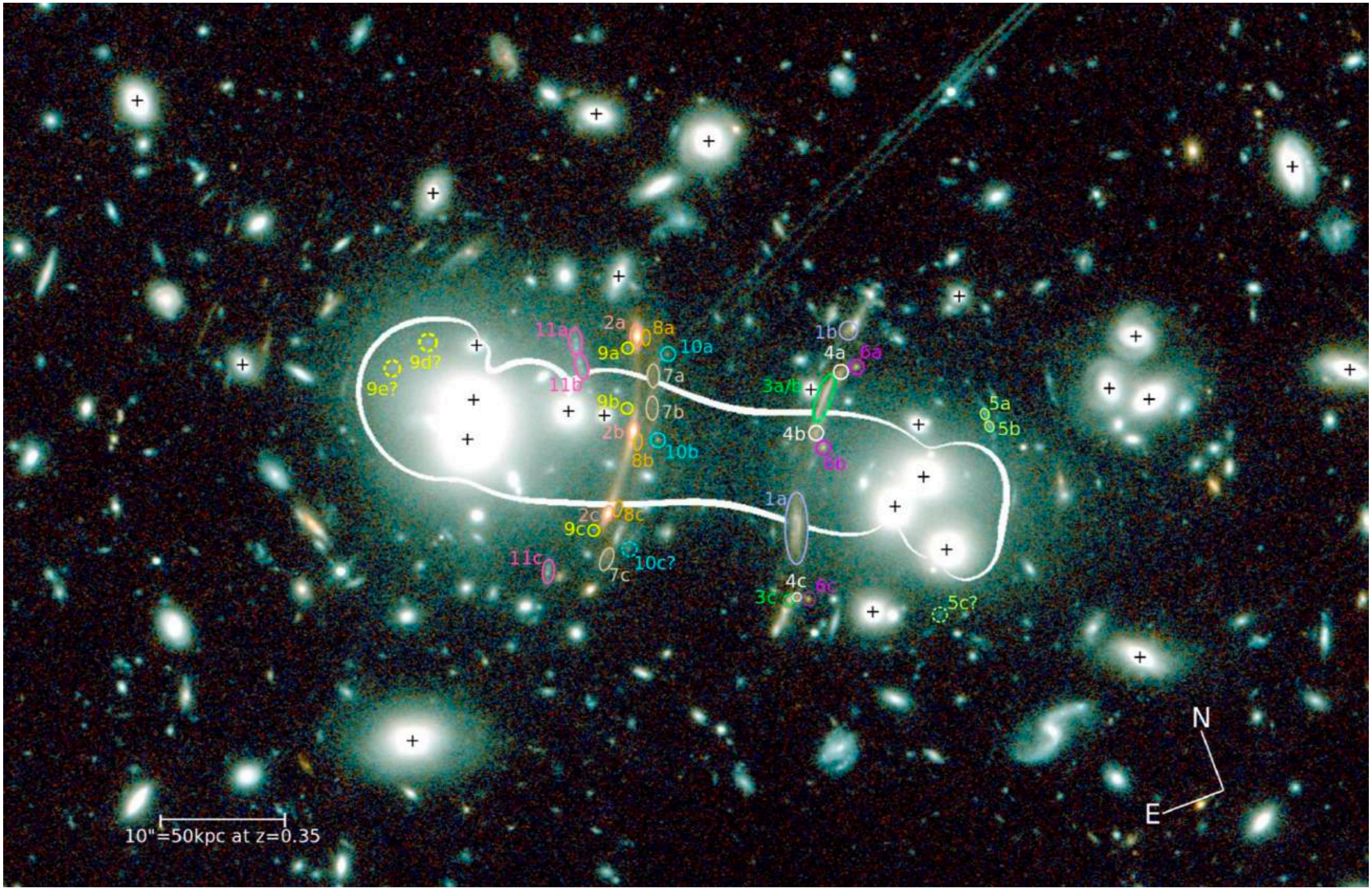


(Frye et al. 2002, 2007, 2008, 2012)

G165 with critical curve and arclet families overlaid: this image was reduced, and the lens model fit made, by Massimo!



(Frye, Pascale et al. 2018)

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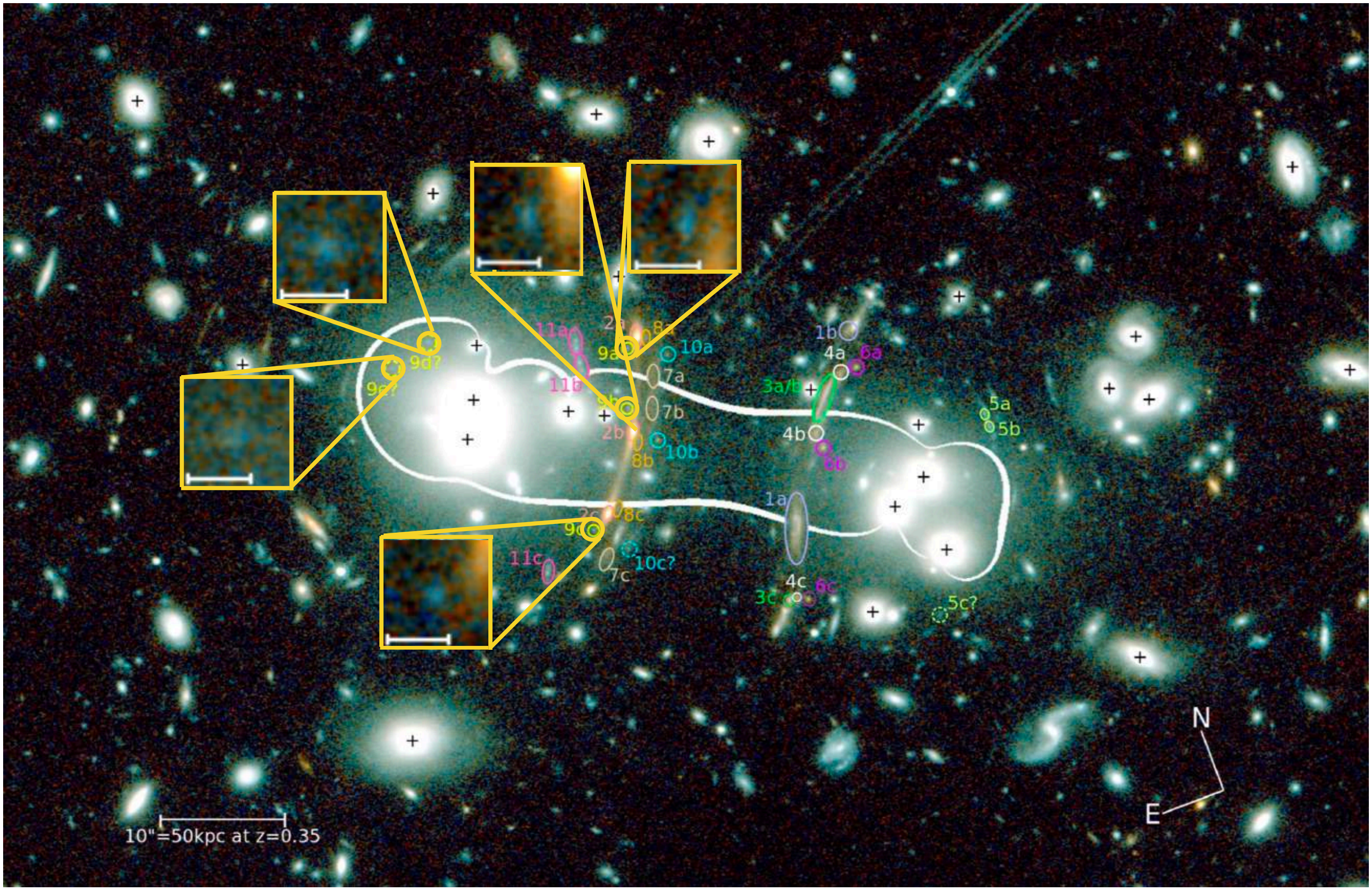


(Frye, Pascale et al. 2018)

Part 2: Lensing by distributed masses - Subhalo

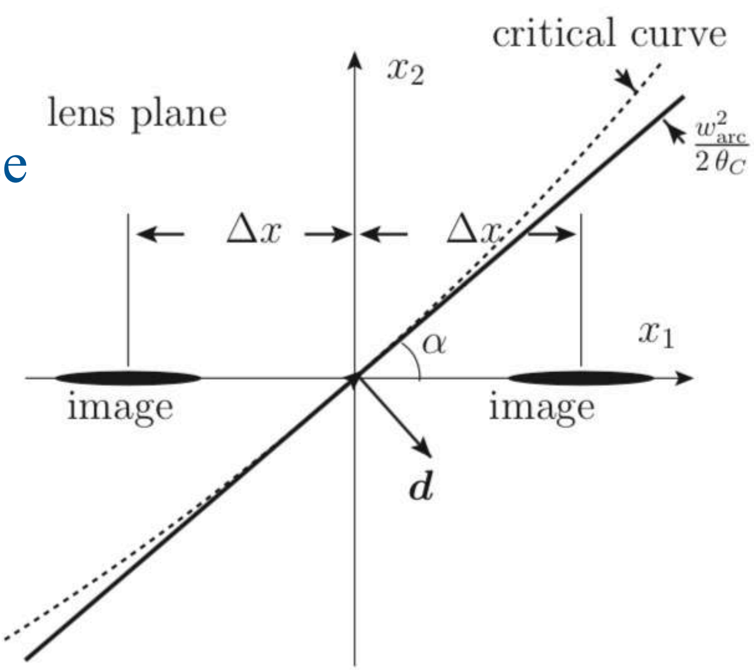
- Dark matter is an outstanding problem: where/what is it? Isolated masses with $M = 10^7$ - 10^8 Msun have $\Sigma \ll \Sigma_{cr}$

- In a cluster, we are given the position of a lensed image $\vec{x}$ whose coordinates are altered from its source coordinates $\vec{y}$: $\vec{y} = A\vec{x}$

- How do we envision this mapping, as α depends on θ nonlinearly? We zoom to vicinity of point x. The best-fit linear map is the Jacobian

- We choose coordinates to be eigenvectors, which means also that A is diagonal. We impose the condition that on the critical curve det(A) = 0. We approximate A:

$$A = \begin{vmatrix} \vec{d}\cdot\vec{x} & 0 \\ 0 & 2(1-\kappa_0) \end{vmatrix}$$



 (Venumadhav et al. 2018; Dai et al. 2018)

Part 2: Lensing by distributed masses - Subhalo

- Recall the cluster lens equation: $\vec{y} = \vec{x} - \vec{\alpha}_B(\vec{x})$ Substituting into A:

$$A = \left[\frac{\partial \vec{y}}{\partial \vec{x}}\right] = \left[\frac{\partial(\vec{x} - \vec{\alpha}_B(\vec{x}))}{\partial \vec{x}}\right] = 1 - \left[\frac{\partial \vec{\alpha}_B(\vec{x})}{\partial \vec{x}}\right]$$

- Let's introduce a subhalo which causes a deflection angle $\vec{\alpha}_{sh}$ which shifts the image to a new position of $\vec{x}' = \vec{x} + \Delta x$. With a subhalo:

$$\vec{y} = \vec{x}' - \vec{\alpha}_B(\vec{x}') - \vec{\alpha}_{sh}(\vec{x})$$

- In each case, the y-vector is is the same. On setting the two lens equations equal to each other and Taylor-expanding the term $\vec{\alpha}_B(\vec{x} + \Delta x)$

$$\vec{x}' - \vec{x} - (\vec{\alpha}_B(\vec{x}') - \vec{\alpha}_B(\vec{x})) = \vec{\alpha}_{sh}$$

$$\Delta x \left(1 - \left[\frac{\partial \vec{\alpha_B}}{\partial \vec{x}}\right]\right) = \vec{\alpha}_{sh}$$

$$\Delta x = A^{-1}(\vec{x}')\vec{\alpha}_{sh}$$

Part 2: Lensing by distributed masses - Subhalo

- The shift in position as a result of the subhalo is: $\Delta x = A^{-1}(\vec{x}')\vec{\alpha}_{sh}$

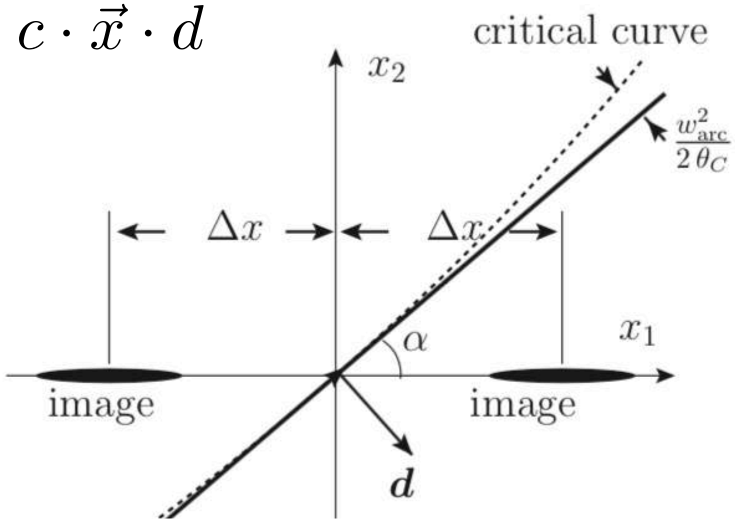
- Recall also the Jacobian: $A^{-1} = \begin{vmatrix} \frac{1}{\vec{d}\cdot\vec{x}} & 0 \\ 0 & 1/c \end{vmatrix}$, where $c = 2(1-\kappa_0)$

- The magnification factor: $\mu = 1/det(A) = \frac{1}{c \cdot \vec{x} \cdot \vec{d}}$

- The shifts in the x1 and x2 directions are:

$$\Delta x_1 = \frac{1}{\vec{x} \cdot \vec{d}} \cdot \alpha_{1,sh}(x') = C\mu\alpha_{1,sh}(x')$$

$$\Delta x_2 = \frac{1}{C}\alpha_{2,sh}(x')$$



- The image is stretched in x_1 direction, and is a double. Compact source images found close to main critical curve identifies subhalos!

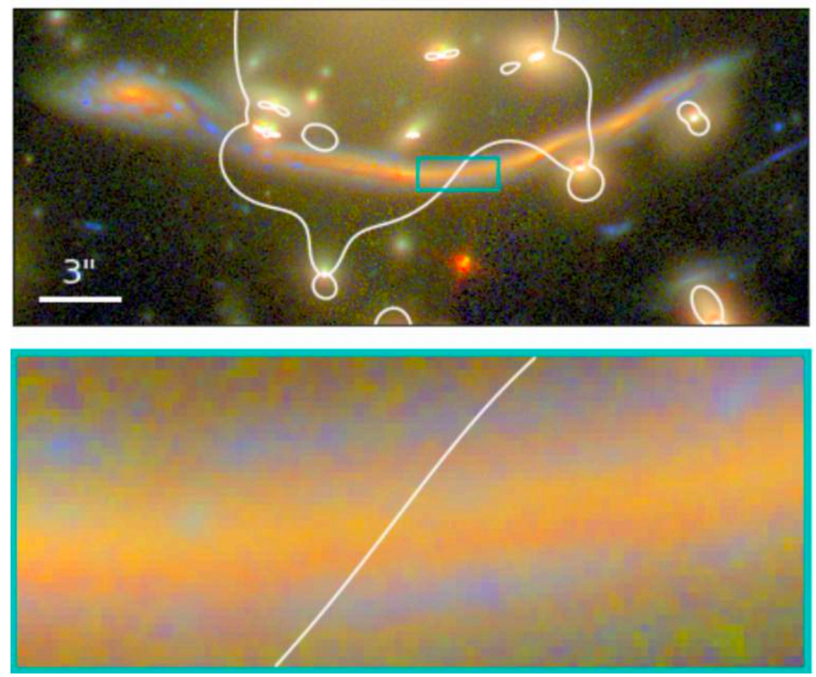
Part 2: Lensing by distributed masses - Subhalo

- An image pair enhanced by subhalo will be stretched along x_1 direction, bridging the critical curve.
- Source must be compact, as a galaxy is too extended.
- Images must be local, ~0.1 arcsec from critical curve!

Arc 1a in G165

(Frye et al. 2018)

The Dragon in A370



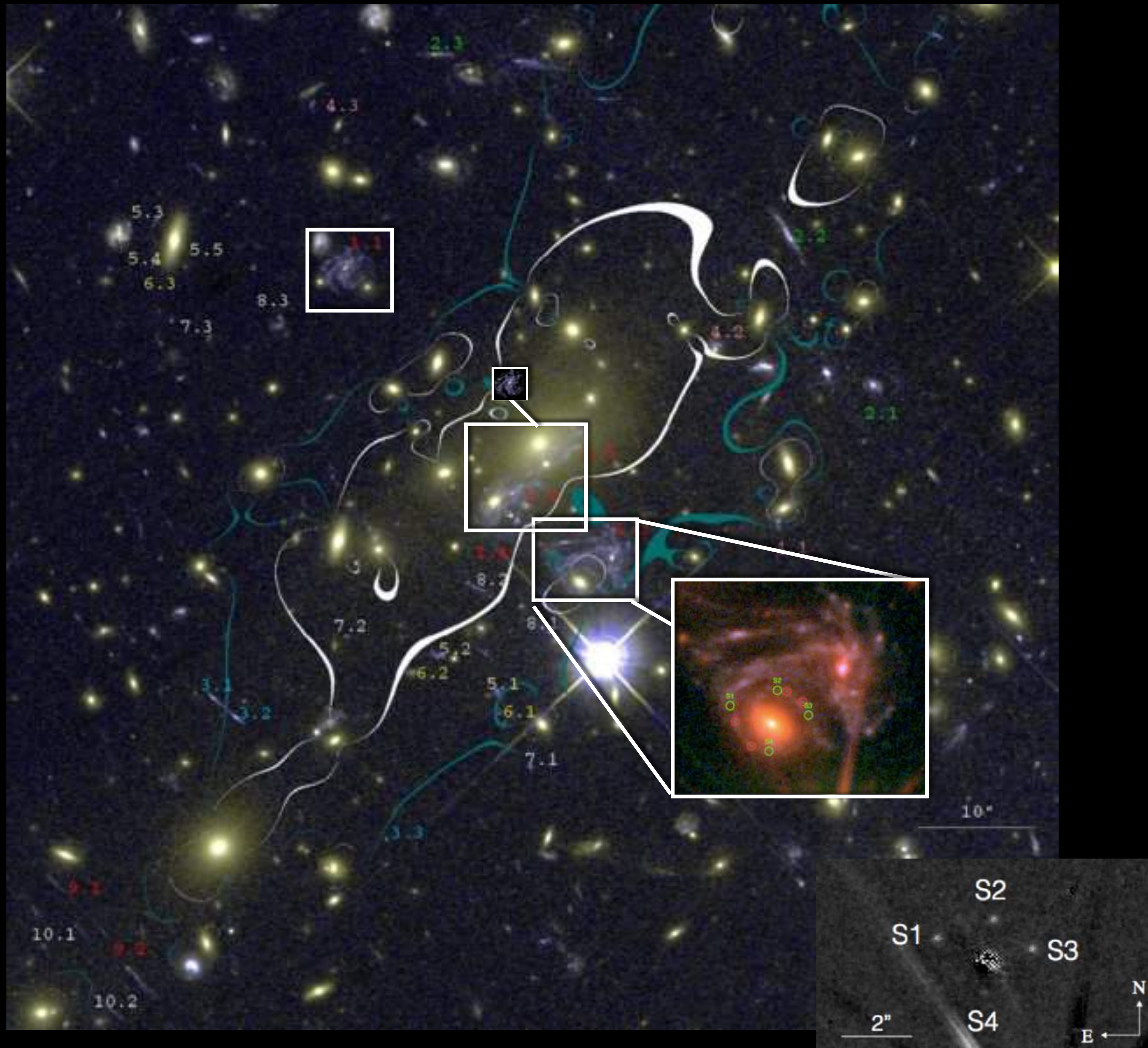
(Dai et al. 2018)

- The "dragon" in A370 is good for its extensive "tail" crossing critical curve
- "Arc 1a" in G165 is an SMG, and may already have blue compact star forming knots. Is this an image pair?

MACS J1149
SN Refsdel,
$z = 1.52$

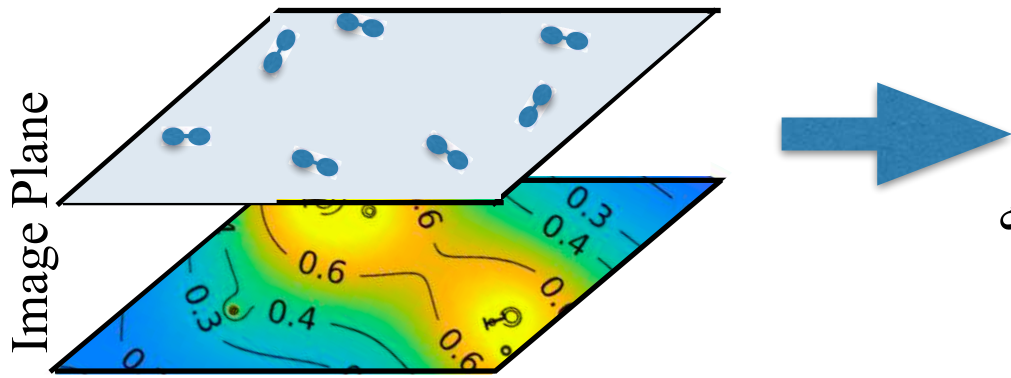
Galaxy host appears in 4 different locations!

SN Refsdel appears in 4 locations in the brightest image, $m_{AB} = 25.5$ mag

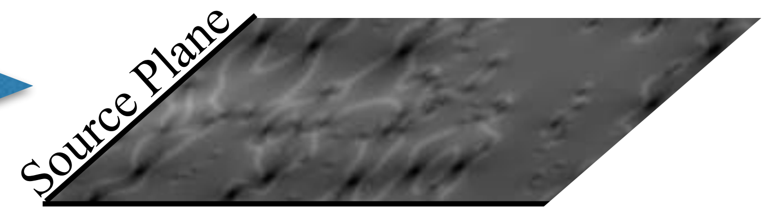


(Kelly et al. 2016, 2018)

Applications: transients



(Frye et al. 2018)

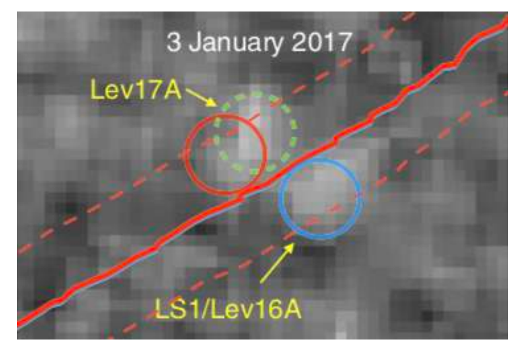


(Diego et al. 2018a)

- Why search for strongly-lensed SNe?
  - standard candles offer independent test of image magnification
  - multiply-imaged SNe count as arclet families for cosmography
  - for ideal geometries, time delays ==> H_0

- Why search for caustic transit events?
  - ultra-high magnification factors of 10^3-10^4
  - sensitivity to microlensing effects
  - route to find first light sources, although event rates depend on ICL, background population, and luck

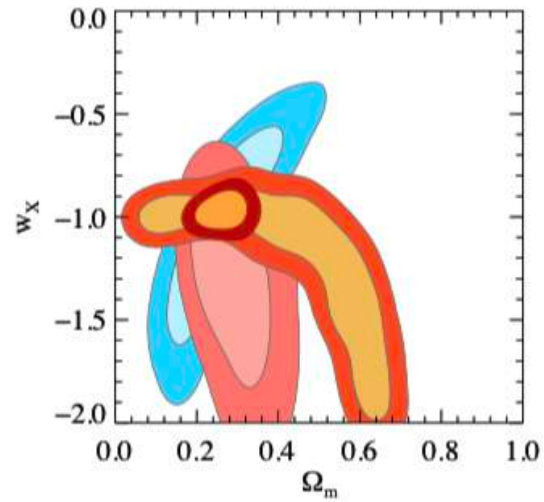
Caustic transit at z=1.53



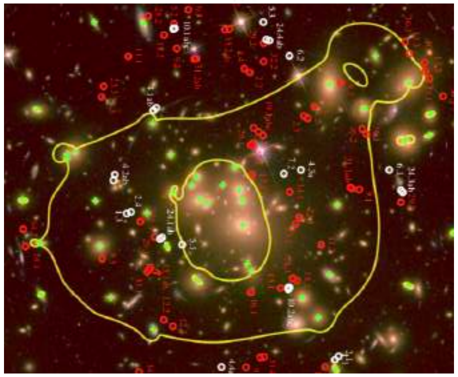
(Kelly+18)

Applications: cosmography

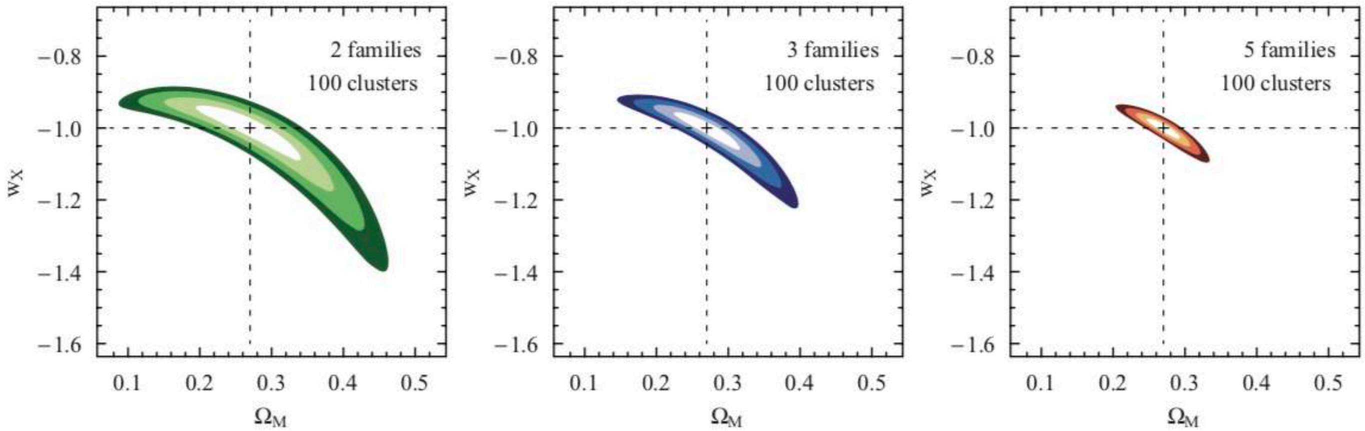
- Constraining Cosmological Parameters, w_x
- Multiple sets of arclet families sensitive to the geometry ==> ratio of angular diameter distances



- Take away**: 2-3 arclet families in 100 clusters with redshifts and lens models yield constraints competitive with other methods, and samples different parameter space**



(Jullo+10)



(Gilmore & Natarajan+09)

Long Duration Transients with LSST: Getting Acquainted with our Dim Neighbors

✦ **Verification** - of 20,000 annual events, 1000s will be degenerate objects. LSST will help break mass/velocity degeneracy that turns measurement t_{cross} into measurements of *mass*

✦ For α ~ 0.1-few, LSST+GAIA+Pan-STARRS enable detection of BH-BH binaries at physical separations are extensions of the ones that will merge to produce LIGO events

✦ LSST annual e**vent rates**[1]: 14,000 M dwarfs, 2,600 WDs, 1,800 neutron stars, 120 BHs, RR Lyrae/other objects.

✦ On WFIRST: which are lessons learned from space-based studies?

$2\theta_E = 0.06''$

$\omega = 0.2'' yr^{-1}$

$\tau_c = 100 days$

(Stellar BH)

[1]from Di Stefano 2008a, consistent with MACHO event rate extrapolation