Assignment 6

Astronomy 541

Assignment: Due Friday, Nov 30

For the first two problems, you can use the fact that the comoving distance r between us and $z = 3$ is c/H_0 in $\Omega_m = 1$ and $1.48c/H_0$ in $\Omega_m = 0.3$ ΛCDM. Similarly, the growth function between us and $z = 3$ is 4 in $\Omega_m = 1$ and 3.13 in the low-density case. From $z = 1$, the growth is 2 and 1.63, respectively. For these problems, use the ΛCDM case except where noted.

In the ΛCDM cosmology (e.g. from Planck), the rms overdensity σ_8 on scales of $8h^{-1}$ Mpc is about 0.8 today. On scales of $1.42h^{-1}$ Mpc ($10^{12}h^{-1}$ $M_\odot$), the overdensities are 2.3 today. On the $8h^{-1}$ Mpc scale, the effective spectral index $n_{\rm eff}$ is about -1.5, while on the smaller scale, $n_{\rm eff} \approx -2.0$. Recall that $n_{\rm eff}$ is defined so that $\sigma_M \propto M^{-(n_{\rm eff}+3)/6}$.

Press-Schechter formalism states that the comoving density of halos is

$$\frac{dn}{d\log M} = \sqrt{\frac{2}{\pi}}\frac{\rho_m}{M}\left|\frac{n_{\rm eff}+3}{6}\right|\nu e^{-\nu^2/2}$$

with $\nu = 1.69/\sigma_M$, where ρ_m is the present-day density of matter ($= \Omega_m\rho_{crit}$).

You don't need to specify a Hubble constant for these problems if you leave masses in h^{-1} $M_\odot$ units and lengths in h^{-1} Mpc units. In these units, the critical density $\rho_{crit} = 2.78\times10^{11}h^2$ $M_\odot$ Mpc^{-3}.

Problem 1 (7 pts): Cosmology from cluster counts: Because of the exponential cutoff in the Press-Schechter mass function, objects with $\nu \gg 1$ have number densities that are very sensitive to the exact value of ν and hence to σ.

With low-redshift clusters of galaxies, we believe that we can estimate their masses fairly well (more on this in a few weeks). Let's say that we have gathered a complete sample of clusters down to a mass threshold of $5\times10^{14}h^{-1}$ $M_\odot$ (to be set equal to the M in the Press-Schechter formalism) in some volume.

a) For the ΛCDM cosmology described above, what is the number density of clusters per logarithmic mass at this mass scale? Note: To compute ν, you will need to compute σ on this mass scale. The easiest way is to convert $8h^{-1}$ Mpc to a mass scale, and then scale σ in mass from σ_8 using the appropriate $n_{\rm eff}$.

b) Now imagine reducing the normalization of the power spectrum by 20%, so that σ_8 is decreased by 10%. How much does this change the number density of clusters?

This sensitivity to cluster counts has proved an effective way to normalize the power spectrum.

c) Let's now compare ΛCDM to $\Omega_m = 1$ Einstein-de Sitter. How must we change the normalization σ_8 to keep the Press-Schechter prediction of the number densities constant at fixed M? In other words, if we have $\sigma_8 = 0.8$ in the ΛCDM cosmology, what must be the value of σ_8 in the Einstein-de Sitter case to match the observed number density?

The value of Ω_m enters in two places: the density ρ_m in the Press-Schechter formula to get σ, and the conversion from σ on mass scale M to σ_8. There is no closed form solution for σ; you will need to compute this numerically (or by trial-and-error with a calculator).

Your answer (which could be used in the next part) should be close to 0.5.

d) Next, imagine that you construct a sample of clusters at $z = 1$ with the same mass threshold of $5 \times 10^{14} h^{-1}$ $M_\odot$. What is the predicted comoving number density of these objects in the ΛCDM cosmology? In Einstein-de Sitter? By scanning volumes of $\sim 10^{10} h^{-3}$ Mpc^3, a sizeable number of $z \sim 1$ clusters have been observed. What can you conclude from this?

Problem 2 (7 pts): LBG halo masses: Lyman break galaxies (LBGs) at $z \approx 3$ are observed to have a number density about 10% of that of galaxies today. They are also observed to be significantly clustered, with a rms overdensity in spheres of $8h^{-1}$ comoving Mpc of about 1.0. In this problem we will use Press-Schechter formalism to estimate the mass of the dark matter halos that LBGs live in. Assume ΛCDM with $\Omega_m = 0.3$ throughout this problem.

a) LBGs are observed to have a density on the sky of 0.4 per square arcminute in a redshift shell of $\Delta z = 0.5$. Compute the number density, in units of comoving h^3 Mpc^{-3}. It is sufficient to get the volume by $(dV/dz)\Delta z$.

b) Estimate the typical mass of halo that contains an LBG, assuming that all halos down to that mass contain a single LBG, and halos below that mass do not contain an LBG. In effect, this is a maximum possible halo mass for LBGs, since there is probably not such a monatonic one-to-one relation between halo mass and LBGs.

To do this, you may need to iterate a bit: solve for M holding ν constant, then compute ν for that mass (scaling from the number above for $1 \times 10^{12} h^{-1}$ $M_\odot$ and including the growth function), and repeat to convergence. At our level of accuracy, it is enough to treat the number density as coming from one e-folding in mass and to neglect integrating over the more massive halos that might contain more than one galaxy.

c) An independent way of estimating LBG halo masses is through their clustering. Halos of a given mass have a given level of clustering, corresponding to a *bias* relative to the overall matter distribution, where bias $b = \sigma_{\rm LBG}/\sigma_{\rm matter}$. Simple models relate bias to ν through $b = (\nu^2 - 1)/1.69 + 1$. Taking b to be the ratio of the observed variance on the $8h^{-1}$ Mpc scale and the overall matter variance on that scale (i.e. σ_8 at $z = 3$), compute the value of ν assuming that all of the galaxies are drawn from a single mass scale of halo.

Convert this value of ν to a mass scale. How does this compare to the answer in part (a)? Factors of two in mass are a success in this game; however, factors of two in clustering strength or bias are not good.

d) The halo mass scale of LBGs could be smaller if the fraction of halos containing an LBG is small. A popular early model postulated that LBGs are not always ‘on’, but instead have some duty cycle during which they vigorously form stars and hence are observable as LBGs. If the duty cycle were 1%, then the number density of host halos must be 100 times larger.

Repeat part (b) with this density to get a mass scale, then use the formula in part (c) to get a predicted bias for LBGs in this scenario. Compare this with the observed bias from clustering.

How badly does this prediction fare? What can you conclude about LBG halo masses?

Problem 3 (6 pts): In this problem we will compute halo cooling radii. Consider a halo with mass M_{200}. Assume that it contains purely pressure-supported hot gas with a density profile following NFW, (i.e., we will ignore the pressure support provided by the hot gas):

$$\rho(r) = \frac{\rho_s r_s^3}{r(r+r_s)^2}$$

with a concentration defined as $c = r_{200}/r_s$.

The gas temperature is everywhere given by the halo virial temperature; this is not exactly true for NFW, but we will ignore the deviations. Assume $\Omega_m = 0.3$, $\Omega_b = 0.045$ and $h = 0.7$, and that the halo contains its cosmic share of baryons.

a) What is the virial temperature T of the halo in terms of M_{200}? Compute T for a $10^{15} M_\odot$ halo.

b) What is ρ_s in terms of c and M_{200}? From this, derive the electron density $n_{e,s}$ at r_s for a fully ionized H plasma (ignore He or metals). Compute $n_{e,s}$ for a halo with $c = 10$.

c) Derive an implicit formula for the cooling radius r_c as a fraction of the virial radius r_{200} (i.e. r_c/r_{200}), in terms of the Hubble time, $n_{e,s}$, c, T (or M_{200}), and the cooling rate Λ.

d) Now let's put all this together: Consider a $10^{15} M_\odot$ halo with $c = 10$ at $z = 0$. What is the cooling radius as a fraction of the virial radius? For the cooling rate, assume purely free-free emission having $\Lambda = 1.7 \times 10^{-27} T^{1/2}$ erg cm^3 s^{-1}. It is sufficient to solve the implicit equation for r_c/r_{200} by trial-and-error.

e) Do the same for a $10^{14} M_\odot$ halo (also with $c = 10$). Does this smaller halo cool out a greater or lesser fraction of its hot gas in a Hubble time?

Problem 4 (10 pts): In a warm ionized hydrogen plasma, the rate per unit volume of recombinations is $\alpha n_e n_i$ where $\alpha \approx 4.2 \times 10^{-13} T_4^{-0.7}$ cm^3 s^{-1}, n_e is the number of free electrons, n_i is the number of free protons, and T_4 is the gas temperature in units of 10^4 Kelvin.

When exposed to a bath of UV photons with energies above 13.6 eV, the ionization rate per unit volume is $\Gamma_{HI} n_{HI}$, where n_{H1} is the number of neutral hydrogen atoms and $\Gamma_{HI} \approx 4.3 \times 10^{-12} J_{21}$ s^{-1}. Here, J_{21} is the specific intensity per unit frequency of the UV spectrum in units of 10^{-21} ergs cm^{-2} s^{-1} sr^{-1} Hz^{-1} as averaged over the ionization cross-section of hydrogen (i.e. one is approximately to evaluate the J_ν of the UV spectrum at a frequency just above 13.6 eV; don't worry about this detail). Remember that converting a specific intensity into the number of photons impinging on a point from an isotropic distribution involves multiplying by a factor of 4π.

a) Show that in the limit of high ionization, the steady-state neutral fraction depends linearly on density and inversely on the incident UV flux.

b) J_{21} has been estimated by adding up contributions from source populations (namely quasars and star forming galaxies); see e.g. Haardt & Madau (1996,2001). Assuming a baryon density of $\Omega_b h^2 = 0.024$ and $J_{21} = 0.3$, compute the neutral fraction at the cosmic mean baryon density, for an IGM temperature of 15,000 K, at $z = 3$.

c) If J_{21} is independent of redshift (which is correct to within a factor of two from $z \sim 4 \rightarrow 1$), what is the redshift dependence of the mean transmission of HI Lyman-α for a purely homogeneous IGM? Be sure to include the redshift dependence of the path length per unit velocity; you may assume matter domination.

d) The cross-section for a hydrogen atom in the ground state to be ionized by a single UV photon of frequency ν is *approximately* $A_0(\nu_1/\nu)^{2.8}$ for $\nu \geq \nu_1$, where $A_0 = 6.3 \times 10^{-18}\mathrm{cm}^2$ and ν_1 is the threshold frequency (i.e. 13.6 eV). This formula is good to about 4% for $1 < \nu/\nu_1 < 4$, but fails at higher energy.

At high redshift, there are fewer sources of ionizing photons and hence J_{21} is lower, though by how much is uncertain. Assuming $J_{21} = 0.1$ at $z = 7$ and $\Omega_b h^2 = 0.024$, compute the mean free path of a UV photon at $z \approx 7$ as a function of energy. For what energy is that mean free path larger than the Hubble distance?

This calculation is interesting, especially at higher redshifts, because it implies that the UV spectrum in the IGM is *harder* than that of the UV sources. In short, at high energies, one can see all the sources in the universe; at low energies, one can only see the nearby ones.