Assignment 6
Astronomy 541

Assignment: Due Tuesday, Nov 25

For the next two problems, you can use the fact that the comoving distance \( r \) between us and \( z = 3 \) is \( c/H_0 \) in \( \Omega_m = 1 \) and \( 1.48c/H_0 \) in \( \Omega_m = 0.3 \) ΛCDM. Similarly, the growth function between us and \( z = 3 \) is 4 in \( \Omega_m = 1 \) and 3.13 in the low-density case. From \( z = 1 \), the growth is 2 and 1.63, respectively. For these problems, use the ΛCDM case except where noted.

In the ΛCDM cosmology (e.g. from WMAP), the rms overdensity \( \sigma_8 \) on scales of 8\( h^{-1} \) Mpc is about 0.8 today. On scales of 1.42\( h^{-1} \) Mpc (10\(^{12}\)\( h^{-1} \) M\(_\odot\)), the overdensities are 2.3 today. On the 8\( h^{-1} \) Mpc scale, the effective spectral index \( n_{\text{eff}} \) is about −1.5, while on the smaller scale, \( n_{\text{eff}} \approx −2.0 \). Recall that \( n_{\text{eff}} \) is defined so that \( \sigma_M \propto M^{-(n_{\text{eff}}+3)/6} \).

Press-Schechter formalism states that the comoving density of halos is

\[
\frac{dn}{d\log M} = \sqrt{\frac{2}{\pi}} \frac{\rho_m}{M} \left| \frac{n_{\text{eff}}+3}{6} \right| \nu e^{-\nu^2/2}
\]

with \( \nu = 1.69/\sigma_M \), where \( \rho_m \) is the present-day density of matter (= \( \Omega_m \rho_{\text{crit}} \)).

You don’t need to specify a Hubble constant for these problems if you leave masses in \( h^{-1} \) M\(_\odot\) units and lengths in \( h^{-1} \) Mpc units. In these units, the critical density \( \rho_{\text{crit}} = 2.78 \times 10^{11} h^2 \text{ M}_\odot \text{ Mpc}^{-3} \).

Problem 1 (7 pts): Cosmology from cluster counts: Because of the exponential cutoff in the Press-Schechter mass function, objects with \( \nu \gg 1 \) have number densities that are very sensitive to the exact value of \( \nu \) and hence to \( \sigma \).

With low-redshift clusters of galaxies, we believe that we can estimate their masses fairly well (more on this in a few weeks). Let’s say that we have gathered a complete sample of clusters down to a mass threshold of \( 5 \times 10^{14} h^{-1} \text{ M}_\odot \) (to be set equal to the \( M \) in the Press-Schechter formalism) in some volume.

a) For the ΛCDM cosmology described above, what is the number density of clusters per logarithmic mass at this mass scale? Note: To compute \( \nu \), you will need to compute \( \sigma \) on this mass scale. The easiest way is to convert 8\( h^{-1} \) Mpc to a mass scale, and then scale \( \sigma \) in mass from \( \sigma_8 \) using the appropriate \( n_{\text{eff}} \).

b) Now imagine reducing the normalization of the power spectrum by 20%, so that \( \sigma_8 \) is decreased by 10%. How much does this change the number density of clusters?

This sensitivity to cluster counts has proved an effective way to normalize the power spectrum.

c) Let’s now compare ΛCDM to \( \Omega_m = 1 \) Einstein-de Sitter. How must we change the normalization \( \sigma_8 \) to keep the Press-Schechter prediction of the number densities constant at fixed \( M \)? In other words, if we have \( \sigma_8 = 0.8 \) in the ΛCDM cosmology, what must be the value of \( \sigma_8 \) in the Einstein-de Sitter case to match the observed number density?
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The value of $\Omega_m$ enters in two places: the density $\rho_m$ in the Press-Schechter formula to get $\sigma$, and the conversion from $\sigma$ on mass scale $M$ to $\sigma_8$. There is no closed form solution for $\sigma$; you will need to compute this numerically (or by trial-and-error with a calculator).

Your answer (which could be used in the next part) should be close to 0.5.

d) Next, imagine that you construct a sample of clusters at $z = 1$ with the same mass threshold of $5 \times 10^{14} h^{-1} M_\odot$. What is the predicted comoving number density of these objects in the $\Lambda$CDM cosmology? In Einstein-de Sitter? By scanning volumes of $\sim 10^{10} h^{-3} \text{Mpc}^3$, a sizeable number of $z \sim 1$ clusters have been observed. What can you conclude from this?

Problem 2 (7 pts): LBG halo masses: Lyman break galaxies (LBGs) at $z \approx 3$ are observed to have a number density about 10% of that of galaxies today. They are also observed to be significantly clustered, with a rms overdensity in spheres of $8 h^{-1}$ comoving Mpc of about 1.0. In this problem we will use Press-Schechter formalism to estimate the mass of the dark matter halos that LBGs live in. Assume $\Lambda$CDM with $\Omega_m = 0.3$ throughout this problem.

a) LBGs are observed to have a density on the sky of 0.4 per square arcminute in a redshift shell of $\Delta z = 0.5$. Compute the number density, in units of comoving $h^3 \text{Mpc}^{-3}$. It is sufficient to get the volume by $(dV/dz) \Delta z$.

b) Estimate the typical mass of halo that contains an LBG, assuming that all halos down to that mass contain a single LBG, and halos below that mass do not contain an LBG. In effect, this is a maximum possible halo mass for LBGs, since there is probably not such a monatonic one-to-one relation between halo mass and LBGs.

To do this, you may need to iterate a bit: solve for $M$ holding $\nu$ constant, then compute $\nu$ for that mass (scaling from the number above for $1 \times 10^{12} h^{-1} M_\odot$ and including the growth function), and repeat to convergence. At our level of accuracy, it is enough to treat the number density as coming from one e-folding in mass and to neglect integrating over the more massive halos that might contain more than one galaxy.

c) An independent way of estimating LBG halo masses is through their clustering. Halos of a given mass have a given level of clustering, corresponding to a bias relative to the overall matter distribution, where bias $b = \sigma_{\text{LBG}}/\sigma_{\text{matter}}$. Simple models relate bias to $\nu$ through $b = (\nu^2 - 1)/1.69 + 1$. Taking $b$ to be the ratio of the observed variance on the $8 h^{-1}$ Mpc scale and the overall matter variance on that scale (i.e. $\sigma_8$ at $z = 3$), compute the value of $\nu$ assuming that all of the galaxies are drawn from a single mass scale of halo.

Convert this value of $\nu$ to a mass scale. How does this compare to the answer in part (a)? Factors of two in mass are a success in this game; however, factors of two in clustering strength or bias are not good.

d) The halo mass scale of LBGs could be smaller if the fraction of halos containing an LBG is small. A popular early model postulated that LBGs are not always ‘on’, but instead have some duty cycle during which they vigorously form stars and hence are observable as LBGs. If the duty cycle were 1%, then the number density of host halos must be 100 times larger.

Repeat part (b) with this density to get a mass scale, then use the formula in part (c) to get a predicted bias for LBGs in this scenario. Compare this with the observed bias from clustering.
How badly does this prediction fare? What can you conclude about LBG halo masses?

**Problem 3 (6 pts):** In this problem we will compute halo cooling radii. Consider a halo with mass $M_{200}$. Assume that it contains purely pressure-supported hot gas with a density profile following NFW, as given in Problem 1 (i.e., we will ignore the pressure support provided by the hot gas). The gas temperature is everywhere given by the halo virial temperature; this is not exactly true for NFW, but we will ignore the deviations. Assume $\Omega_m = 0.3$ and $\Omega_b = 0.045$, and that the halo contains its cosmic share of baryons.

a) What is the virial temperature $T$ of the halo in terms of $M_{200}$? Compute $T$ for a $10^{15}M_\odot$ halo.

b) What is $\rho_s$ in terms of $c$ and $M_{200}$? From this, derive the electron density $n_{e,s}$ at $r_s$ for a fully ionized H plasma (ignore He or metals). Compute $n_{e,s}$ for a halo with $c = 10$.

c) Derive an implicit formula for the cooling radius $r_c$ as a fraction of the virial radius $r_{200}$ (i.e. $r_c/r_{200}$), in terms of the Hubble time, $n_{e,s}$, $c$, $T$ (or $M_{200}$), and the cooling rate $\Lambda$.

d) Now let’s put all this together: Consider a $10^{15}M_\odot$ halo with $c = 10$ at $z = 0$. What is the cooling radius as a fraction of the virial radius? For the cooling rate, assume purely free-free emission having $\Lambda = 1.7 \times 10^{-27}T^{1/2}$ erg cm$^3$ s$^{-1}$. It is sufficient to solve the implicit equation for $r_c/r_{200}$ by trial-and-error.

e) Do the same for a $10^{14}M_\odot$ halo (also with $c = 10$). Does this smaller halo cool out a greater or lesser fraction of its hot gas in a Hubble time?