Final Review
Astronomy 541

1 Basic Format

Final exam will be Wednesday Dec 12 at 1pm. You can either do it in room 204 or bring it back to your office to take it. It is 120 min. This exam is closed book and closed notes. You may use a calculator. I will provide a formula sheet with the exam which shall include the equations and numerical constants that you will need for the exam. However, there are some definition and scaling relation that you should know by heart (e.g., Friedmann equation, virial theorem, Einstein radius). I have included a superset of those equations that we encountered during our class. A good starting point for your review is to understand their meaning. You do not need to memorize the more difficult ones, or how to derive them in detail, but the key is that you should know the physical meanings and applications.

There are 10 questions, worth a total of 100 points. The first six questions require short (but clear) answers and no derivations; the next questions asks you to discussion in one or two short paragraphs about a concept; the last two questions require you to do some derivations. Examples:

Problem 1 (5 pts): Define galaxy power spectrum and its relation with galaxy two-point correlation function.

Problem 2 (10 pts): Describe thermal S-Z effect and its frequency dependence; how to use S-Z effect, combined with X-ray observations, to measure the density and temperature of a cluster and to measure Hubble constant; discuss the main assumptions made in these measurements.

Example quantitative question will be one of the shorter (5 point) homework questions, or half of the long homework questions.

The first two kinds of questions are very similar to those you will encounter in your close-booked prelim; the quantitative one will be similar in complexity for the open-booked prelim questions, although those in prelim tend to be somewhat broader in the area of knowledge they are testing while those in your final only concerns cosmology.

For those of you taking prelim early next semester, please use this as a guide to study prelim as well.
2 General Thoughts

I regard the cosmology class as a class that should focus on the basic concepts, principles, observables and tests in cosmology, with the goal of establishing and testing our basic world model, and apply this basic world model to extragalactic astronomy. Therefore, I encourage you to focus on the big picture concepts, rather than technical details. I think a good indication of whether you are prepared for the final (and prelim) is this:

You should be able to explain the basic cosmological parameters that describe our six-parameter LCDM Planck cosmology – what these parameters are (and the related derived parameters), what their physical meanings are, how we measure them and if there are uncertainties and tensions in either their physical interpretation or measurements, e.g., evidence of dark matter and dark energy.

Our class this semester went through three stages: basic homogeneous cosmology (both matter dominated - classical cosmological tests and radiation dominated - hot big bang); perturbative cosmology (power spectrum, correlation function as related to redshift surveys, and CMB power spectrum); and non-linear collapse (basic processes and scales of gravitationally collapse, galaxy formation and reionization). Your preparation should focus on the key concepts along these three stages, the important physical processes that we have discussed (many involves Zel’dovich...), and how we observe them.

3 Basic Concepts to Review

• **Classical Cosmology:** Importance of homogeneity, isotropy, and expansion of the Universe as the basis for the cosmological model; Metric, and the FRW metric; cosmological redshift; Hubble expansion from metric; cosmological distances, volume and time; horizons; Friedmann equations; Recasting of Friedmann eq as $H(z)$; Interplay of density, curvature, and destiny of the Universe.

• **Cosmological Tests:** Trends of distance, volume, and time for different cosmologies; classical cosmological tests - $H_0$ and density parameters; Distance ladders, direct distance methods, SNe, etc.; Age of the Universe; dark energy: evidence, parameterization

• **Hot Big Bang:** Cosmic microwave background – basic implications; Recombination; Thermal history of the Universe; Big Bang nucleosynthesis – basic pathways, Helium-4, D/H, observations

• **Perturbations and Large Scale Structure:** linear perturbation theory; solution for growth functions; Jeans instability; correlation function, power spectrum; Harrison-Zeldovich power spectrum; $\sigma$ and $\sigma_8$; physics of CDM power spectrum; redshift survey
• **CMB**: Basic physics; Sachs-wolfe effect; Acoustic peaks; Silk damping; What we learn about cosmology from the CMB power spectrum, e.g., parameter dependences; polarization and reionization.

• **Spherical Collapse**: Zel’dovich approximation; Spherical collapse; Virialization and halo scalings; NFW profiles; Press-Schechter and halo masses;

• **First galaxies**: Cooling in galaxies, mass scale, IMF, observations of high-z galaxies, Lyman break galaxy

• **Reionization and IGM**: basic physical processes, observational constraints, Gunn-Peterson effect; Lyman α absorbers; the fluctuating Gunn-Peterson approximation

• **Galaxy Evolution**: The hierarchical galaxy formation cartoon; problems in galaxy formation

• **Gravitational lensing**: lens equation for a point mass; microlensing; weak lensing; singular isothermal sphere. NB: we didn’t talk about it in detail during this course. It is covered in the galaxy class, guest lectures and will be discussed some more next week; however, it leave this here for those of you who are preparing prelim. You can assume that I will not test this explicitly in the final.

• **Clusters**: Basic properties Mass estimation from X-ray gas, lensing, S-Z, velocity dispersion; clusters and cosmology
The following formulae could be of use on the exam. You are responsible for knowing what the symbols mean and when the formulae are applicable.

Please note that $R(t)$ is the same as $a(t)$, the scale factor of the Universe.

- $c = 3.00 \times 10^{10}$ cm/s
- $h = 6.63 \times 10^{-27}$ erg s = $2\pi\hbar$
- $G = 6.67 \times 10^{-8}$ dynes cm$^2$ g$^{-2}$
- $k_B = 1.38 \times 10^{-16}$ erg/K
- $\sigma_{SB} = 5.67 \times 10^{-5}$ erg cm$^{-2}$ s$^{-1}$ K$^{-4}$
- $a_{SB} = 7.56 \times 10^{-15}$ erg cm$^{-3}$ K$^{-4}$
- $m_p = 1.67 \times 10^{-24}$ g
- $1/H_0 = 9.78 h^{-1}$ Gyr
- $c/H_0 = 3000 h^{-1}$ Mpc

\[ ds^2 = c^2 dt^2 - R(t)^2 \left[ dr^2 + S(r)^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) \right], \quad S(r) = \begin{cases} R_c \sin(r/R_c) & \text{if } \kappa = -R_c^2, \\ R_c \sinh(r/R_c) & \text{if } \kappa = 0 \end{cases} \]

\[ ds^2 = dt^2 - \frac{R(t)^2}{c^2} \left[ \frac{dr^2}{1 - \kappa r^2} + r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) \right], \quad \kappa = \pm R_c^{-2} \text{ or } 0 \]

\[ H(t) = \frac{1}{R(t)} \frac{dR}{dt} \]
\[ r(z) = \int_0^z \frac{c \, dz}{H(z)} \]
\[ D_A(z) = \frac{S[r(z)]}{(1 + z)} \]
\[ D_L(z) = S[r(z)](1 + z) \]
\[ f(E) = \frac{1}{\exp[(E - \mu)/kT] + 1} \]
\[ \rho_c = \frac{3H_0^2}{8\pi G} \]
\[ \Omega_m = \frac{\rho_m}{\rho_c} \]
\[ \rho_c^2 = g \int \frac{d^3p}{h^3} f(E)E \]
\[ R = -\frac{4\pi G}{3} R \left( \rho + \frac{3p}{c^2} \right) + \frac{1}{3} \Lambda R \]
\[ \dot{R}^2 = \frac{8\pi G}{3} \rho R^2 - \kappa c^2 + \frac{1}{3} \Lambda R^2 \]
\[ H(z) = H_0 \left[ \Omega_{rad}(1 + z)^4 + \Omega_m(1 + z)^3 + \Omega_\Lambda + (1 - \Omega_{rad} - \Omega_m - \Omega_\Lambda)(1 + z)^2 \right]^{1/2} \]
\[ \rho_c^2 = g_s (a_{SB}/2) T^4 \]
\[ g_s = (\text{Spin states of relativistic bosons}) + \frac{7}{8} (\text{Spin states of relativistic fermions}) \]
\[ S[r(z)] = \frac{2c}{H_0 \Omega_0^2 (1 + z)} \left\{ \Omega_0 z + (\Omega_0 - 2) \left[ (\Omega_0 z + 1)^{1/2} - 1 \right] \right\} \quad (\Lambda = 0) \]
\[
\begin{align*}
\frac{d_\rho}{dt} + \rho \nabla_x \cdot \mathbf{v} &= 0 \\
\frac{d\mathbf{v}}{dt} &= -\frac{1}{\rho} \nabla_x p - \nabla_x \phi \\
\nabla_x^2 \phi &= 4\pi G \rho \\
x &= Rr, \quad \mathbf{v} = Hx + \delta \mathbf{v} = Hx + Ru \\
\frac{d^2 D}{dt^2} + 2H \frac{dD}{dt} &= \left(4\pi G \rho_h - \frac{c_s^2 k^2}{R^2}\right) D \\
\frac{d\delta}{dt} + (\mathbf{u} \cdot \nabla_r) \delta + (1 + \delta) \nabla_r \cdot \mathbf{u} &= 0 \\
\frac{d\mathbf{u}}{dt} + 2H \mathbf{u} + (\mathbf{u} \cdot \nabla_r) \mathbf{u} &= -\frac{c_s^2 \nabla_r \delta}{R^2 (1 + \delta)} - \frac{1}{R^2} \nabla_r \phi \\
\nabla_r^2 \phi &= 4\pi GR^2 \rho_h \delta \\
\delta \mathbf{v} &= \frac{2f}{3H\Omega} \\
f &= \frac{a \frac{dD}{da}}{D} \approx \Omega^{0.6}
\end{align*}
\]

\[
D(t) \propto 1 + \frac{3}{x} + \frac{3\sqrt{1 + x}}{x^{3/2}} \ln \left[ \sqrt{1 + x} - \sqrt{x} \right], \text{where } x = R(t)(1 - \Omega_m)/\Omega_m \text{ and } \Lambda = 0
\]

\[
D(t) \propto (1 + z)^{-1} \text{ for Einstein-de Sitter.}
\]

\[
\sigma^2 = \int \frac{dk}{k} \frac{k^3 P(k)}{2\pi^2} \left| \tilde{W}(k) \right|^2 \text{ where } \tilde{W}(k) = \frac{3kR\cos(kR) - 3\sin(kR)}{(kR)^3} \text{ for spherical top hat}
\]

\[
\rho_{NFW} \propto \frac{1}{r(r + r_s)^2}, \quad r_{200}/r_s = c
\]

\[
\frac{dn}{d\ln M} = \frac{\rho_0}{M} \sqrt{\frac{2}{\pi}} \left| \frac{n + 3}{6} \right| e^{-\nu^2/2}
\]

\[
\frac{dn}{dL} = \frac{1}{L_\star} \left( \frac{L}{L_\star} \right)^\alpha \exp(-L/L_\star)
\]

\[
\Gamma(x) = \int_0^\infty dt \ t^{x-1} e^{-t}
\]

\[
\alpha = \frac{4GM(< b)}{bc^2}
\]

\[
\theta - \frac{dLS}{dS} \alpha = \beta
\]

\[
\beta = \left| \frac{\theta}{\beta} d\beta \right|
\]

\[
s = \int_0^{z_s} c_s(1 + z)dt
\]

\[
\tau_{HI} = A \left( \frac{\rho}{\bar{\rho}} \right)^\beta, \quad \beta \approx 1.6
\]

\[
A = 1.13 \left( \frac{1 + z}{4} \right)^6 \left( \frac{\Omega_b h^2}{0.02} \right)^2 \left( \frac{T_0}{2 \times 10^4 \text{K}} \right)^{-0.7} \left( \frac{\Gamma_{HI}}{10^{-12} \text{ s}^{-1}} \right)^{-1} \left( \frac{H(z)}{312 \text{ km/s/Mpc}} \right)^{-1}
\]