Final Exam Solutions

Astronomy 541

The average of the final exam is 76 with a standard deviation of 14 and a fairly reasonable Gaussian distribution. I would say that if your final falls outside 1-σ lower than this, then you need to work harder on your prelim preparation on this subject; for those below average, you might want to put some extra work as well.

Problem 1 (5 pts): Order the following events in time (1 being earliest, 9 being most recent):

A. Planck Time

B. Electron-positron annihilation

C. Recombination

D. The formation of rich galaxy clusters

E. Hydrogen reionization

F. Formation of the first star

G. Helium production

H. Neutrino decoupling

I. Matter-radiation equality

Answer: A - H - B - G - I - C - F - E - D

Problem 2 (5 pts): Give two observational evidences that there might be a form of dark energy in the universe.

Answer:

- SN Ia observations
- CMB flatness + low-Omega

Problem 3 (5 pts): For most galaxies in the Hubble Deep Field, to what approximate proper length (in kpc) does 1 arcsecond correspond? You do not need to justify your answer.

Answer: about 5 physical kpc.

Problem 4 (5 pts): Define the two-point correlation function that measures galaxy clustering; describe the measurement of two-point correlation function at $z \sim 0$. Is the clustering of rich cluster of galaxies stronger or weaker than that of galaxies?

Answer: explain $\xi(R)$ as excess probability; $\xi(R) \approx \left(\frac{r}{r_0}\right)^{-1.8}$. $r_0 \sim 5h^{-1}$ Mpc. Clusters of galaxies are much more strongly clustered. ($r_0 \sim 20$ Mpc).

Problem 5 (5 pts): In Big Bang nucleosynthesis, the deuterium abundance is regarded as the best measure of the baryon density. What is the sign of the trend of [D/H] versus baryon density? What is the physical explanation for this trend? (one or two brief sentences is sufficient).

Answer: In the late stages of Big Bang nucleosynthesis, deuterium burns almost completely to helium-4. The higher the density of baryons, the more completely this reaction runs, leading to a lower density of deuterium.

Problem 6 (5 pts): What is hydrogen reionization? Roughly when did it happen in the universe? What are the likely energy sources?

Answer: IGM changes from neutral to ionized; at z=6-10, by early galaxies and AGN.

Problem 7 (15 pts):

Essentially all cosmological observations, including CMB, large scale structure and its growth at low-redshift universe, are consistent with the so-called "concordance" ΛCDM model, that the universe is geometrically flat, with cold dark matter and a large cosmological constant. This Λ CDM model can be completely described by only six *independent* cosmological parameters. What are these parameters? Note that some of the common cosmological parameters are not completely independent, e.g, the age of the universe can be written as a combination of Hubble constant and density parameter in a flat universe. List only independent ones. Note that we are already assuming the universe is flat, so curvature is not an independent parameter in this case.

Answer: (no unique answer, but some combination of) expansion rate, baryon density, matter density, shape of power spectrum, amplitude of power spectrum, and reionization optical depth.

Problem 8 (15 pts): Draw (approximately) a CMB power spectrum. Then describe the main physical processes that are responsible for the large scales ($l < 100$), intermediate scales ($100 < l < 1000$), and small scales ($l > 1000$). What cosmological parameter is the dominant factor that determine the location of the first acoustic peak?

Answer: y-axis is $C_l l(l+1)$; large scale: Sachs-Wolfe; intermediate scale: acoustic peaks; small scale: statistical and silk damping, S-Z, discrete sources. Location of the first peak is almost entirely determined by Ω_k.

Problem 9 (15 pts): We approximate the virial radius of a halo as the radius inside of which the density is 200 times the mean density of the Universe. We call this radius R_{200}. The mass inside this radius is M_{200} and the circular velocity at this radius is V_{200}, assuming spherical symmetry. Your answers to the following should include the dependences on Ω_m and H_0.

a) Derive a relation between M_{200} and V_{200} for a spherical halo. R_{200} should not appear.

Answer: The mean density is given by $\bar{\rho} = \rho_c \Omega_m = 3H_0^2\Omega_m/(8\pi G)$. Hence the mass enclosed within the virial radius is

$$M_{200} = \frac{4\pi}{3} 200 \bar{\rho} R_{200}^3 = 100 H_0^2 \Omega_m R_{200}^3 / G.$$

To relate R_{200} to V_{200} we use the virial theorem, which states that $T = -0.5V$, where T, V are the kinetic and potential energies. Hence (to within factors of order unity)

$$\frac{1}{2} V_{200}^2 = \frac{GM_{200}}{2R_{200}} \quad \Longrightarrow \quad R_{200} = \frac{GM_{200}}{V_{200}^2}.$$

Pluggin this back into the formula for M_{200} and solving for M_{200}, we have

$$M_{200} = \frac{1}{10 G H_0 \sqrt{\Omega_m}} V_{200}^3.$$

b) Defining the crossing time as R_{200}/V_{200}, derive a relation between the crossing time and the Hubble time.

Answer: The Hubble time $t_H \approx H_0^{-1}$. So

$$\frac{R_{200}}{V_{200}} = \frac{GM_{200}}{V_{200}^3} = \frac{0.1}{\sqrt{\Omega_m}} t_H$$

Problem 10 (25 pts): Recently a gamma-ray burst was observed at redshift 8. Gamma ray bursts are so bright that they can be seen out to enormous redshift, let's say $z = 24$.

a) In an $\Omega_m = 1$ universe, if we assume that the burst is equally likely to come from any point in the observable universe (i.e., equally distributed in comoving volume), what fraction of bursts should come from $8 < z < 24$ (assuming that all come from $z < 24$)?

Answer: The volume per steradian inside a given redshift in a flat cosmology is

$$V(< z) = \int_0^z \frac{dV}{dz} dz = \int_0^z \frac{r(z)^2 c}{H(z)} dz = \frac{r(z)^3}{3}$$

using $dr/dz = c/H$ (you did a similar manipulation in problem set 2). Then the fraction of bursts we want is $1 - V(z < 8)/V(z < 24) = 1 - r(8)^3/r(24)^3$. For $\Omega_m = 1$, we have

$$r(z) = \frac{2c}{H_0} \left(1 - \frac{1}{\sqrt{1+z}} \right).$$

We find that 42% of gamma-ray bursts would come from $z > 8$ in this model.

b) Instead, if we assume that bursts are produced at constant rate per unit time and per comoving volume, write a formula for the fraction of bursts coming from $8 < z < 24$. This formula should be good for an arbitrary cosmology and be written with integrals over redshift with an integrand involving $S[r(z)]$, $H(z)$, and $1 + z$. You do not need to evaluate the integrals!

Answer: Again we want to integrate over volume, but now we need to weight the volume differently. If the bursts occur at a fixed rate in time, then we will receive them at a rate that is decreased by time dilation by a factor of $1 + z$. So the answer is

$$f = \frac{\int_8^{24} \frac{dV}{dz} \frac{dz}{1+z}}{\int_0^{24} \frac{dV}{dz} \frac{dz}{1+z}} = \frac{\int_8^{24} \frac{S[r(z)]^2 c}{H(z)} \frac{dz}{1+z}}{\int_0^{24} \frac{S[r(z)]^2 c}{H(z)} \frac{dz}{1+z}}$$