Hints for Assignment 2

Astronomy 541

Problem 1:

For $r(z)$, open and lambda universes have a larger coordinate distance to a given redshift than does a critical density universe. Both of these universes hence have a larger angular diameter distance, with the extra boost from curvature in an open universe slighly compensating the extra $r(z)$ in a lambda universe. This trend is exactly mimicked in the distance modulus, of course.

For comoving volume, lambda universes have far more volume per square degree per redshift at $z \approx 1$, with open universes catching up at higher redshifts.

Lambda universes have larger lookback times for a given Hubble constant, followed by open universes. Critical-density universes are the youngest, smallest, and nearest.

For reference in Problems 2–4, in the $\Omega = 1$ cosmology, $H(1.75) = 4.56H_0$ and $D(r(1.75)) = 2382h^{-1}$ Mpc. In the $\Omega = 0.3$ open cosmology, $H(1.75) = 3.40H_0$ and $D(r(1.75)) = 3046h^{-1}$ Mpc. In the $\Omega = 0.3$ cosmology with a cosmological constant, $H(1.75) = 2.63H_0$ and $D(r(1.75)) = 3360h^{-1}$ Mpc.

Note: in **open** model, $d_A = R_c sinh(r/R_c)/(1+z)$, not sin!

Problem 2:

$D(r) = R_c sinh(r/R_c)$

The luminosity distance to the $z = 1.75$ galaxy is $(1+z)D(r) = 2.75D(r(1.75))$. The luminosity distance to the nearby object is 7 Mpc.

By our assumption about the bandpass, the photons emitted in the rest-frame R band of the distant galaxy are all received in the K band. Their flux has been reduced by a factor of the ratio of the square of the two luminosity distances.

However(!), the zeropoints of the magnitude systems are given in terms of flux per unit frequency. The range of frequencies of the distant light has been shrunk by a factor of $1 + z = 2.75$. This means that an equivalent amount of flux actually corresponds to a flux density that is 2.75 times *higher*.

The R band luminosity per unit frequency of the nearby source is 3080 Janskies times 10^{-4} (because $R = 10$) times $4\pi d_L^2$. In total, this is $L_\nu = 1.8 \times 10^{28}$ ergs/s/Hz.

Hence, for $\Omega = 1$, the flux per unit frequency in K should be $[L_\nu/4\pi d_L(1.75)^2] \times (1+z)$, which is 4.75×10^{-30} ergs/cm^2/s/Hz or 4.75×10^{-7} Janskies. Comparing to the zeropoint of the K band, this is $K = 22.82$.

The other cosmologies scale simply by the ratios of the square of the luminosity distances. The open universe gives $K = 23.36$; the lambda universe gives $K = 23.57$.

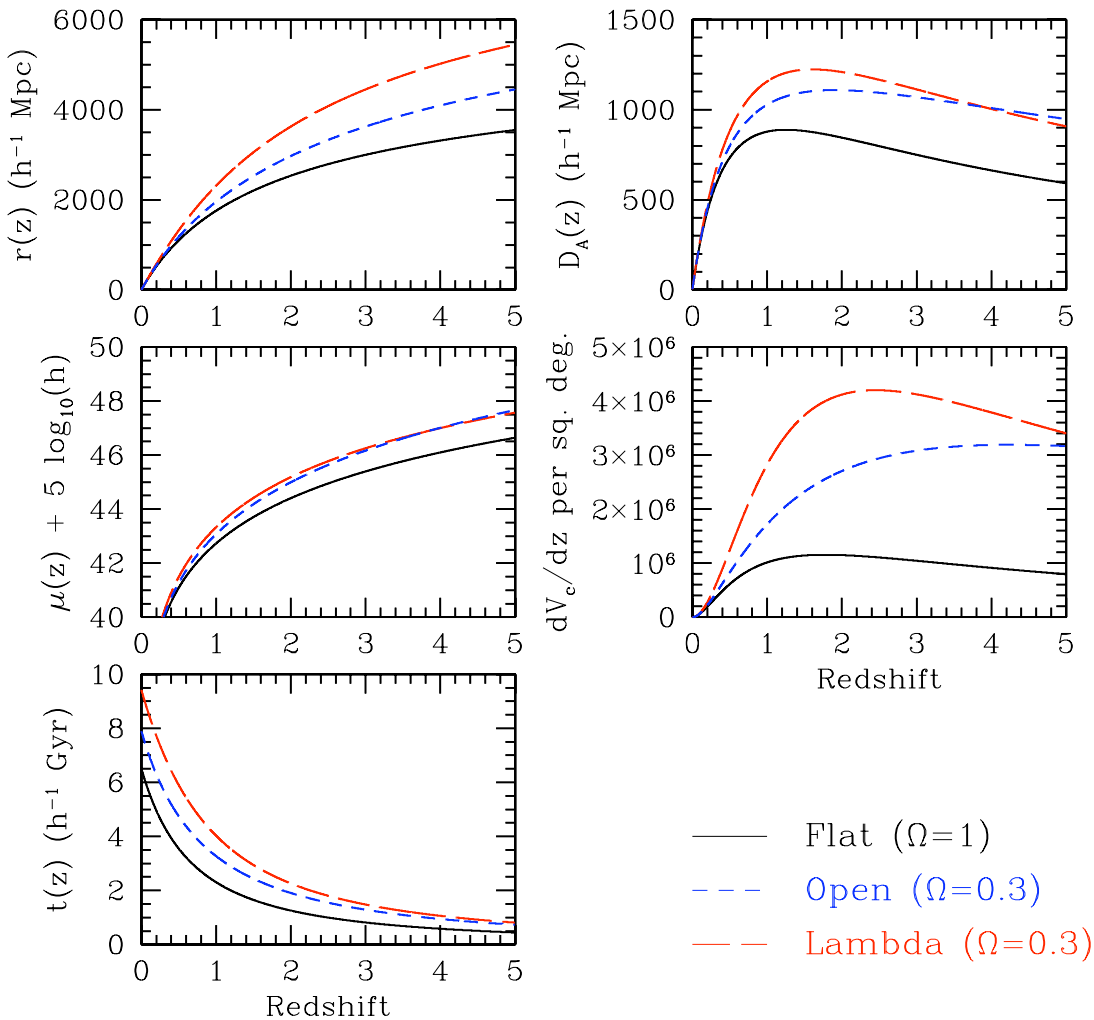

Figure 1: figure for Problem 1

Problem 3 The comoving volume per steradian per unit redshift is $dV/dz\,d\Omega = D(r)^2 c/H(z)$. Hence, the comoving volume per square degree per 0.1 in redshift is

$$V_c = D(r)^2 \frac{c}{H(z)}\ \mathrm{Mpc} \frac{0.1}{3282.8\ \mathrm{deg\ sr}^{-1}} \tag{1}$$

which is 3.31×10^5 Mpc^3 in the $\Omega = 1$ universe.

We are given a number density of galaxies of 0.01 Mpc^{-3} in the present day. If the galaxies aren't changing, then their number per comoving volume is fixed, and hence the above number density is their comoving number density. This means that there are 3314 galaxies per square degree between $z = 1.7$ and 1.8.

In the open cosmology, the number is 7279. In the lambda cosmology, the number is 11417.

Problem 4:

The objects are close enough on the sky that we can calculate the distance as the Pythagorean sum of the distance along the line of sight and the distance transverse to the line of sight.

The transverse distance is just the angular diameter distance times the angular separation, or

$$\ell_a = D_A(1.75) \frac{40''}{206000''\ \mathrm{rad}^{-1}} = \frac{D(r(1.75))}{2.75}\ \frac{40}{206000} = 7.06 \times 10^{-5} D(r(1.75)) \tag{2}$$

The distance along the line of sight can be computed from $(dr/dz)\Delta z$. However, the resulting Δr is a comoving distance, because r is the comoving radial coordinate. Hence, the physical distance along the line of sight is

$$\ell_r = \frac{dr}{dz}\frac{1}{1+z}\Delta z = \frac{c}{H(z)(1+z)}\Delta z = \frac{3000h^{-1}\ \mathrm{Mpc} \times 0.003}{2.75 H(z)/H_0} = \frac{3.27h^{-1}\ \mathrm{Mpc}}{H(1.75)/H_0} \tag{3}$$

In the $\Omega = 1$ universe, $\ell_a = 168h^{-1}$ kpc, $\ell_r = 718h^{-1}$ kpc, and the total separation is 1.05 Mpc.

In the open universe, $\ell_a = 215h^{-1}$ kpc, $\ell_r = 963h^{-1}$ kpc, and the total separation is 1.41 Mpc.

In the lambda universe, $\ell_a = 237h^{-1}$ kpc, $\ell_r = 1244h^{-1}$ kpc, and the total separation is 1.81 Mpc.

Problem 5 (3 pts): If the universe is at least 12 Gyr old, then if $\Omega = 1$ we must have $H_0 < 2/3t$, which is $h < 0.54$ (recalling $H_0^{-1} = 9.78h^{-1}$ Gyr).

For the open model with $\Omega = 0.2$, we have $t_0 = 0.846 H_0^{-1}$, so $h < 0.69$.

For the lambda model with $\Omega = 0.2$, we have $t_0 = 1.08 H_0^{-1}$ so $h < 0.88$.