1 Gunn-Peterson Effect

First detection in 60’s with Maarten Schmidt’s observations of fast-receding point sources called “quasi-stellar objects”. If recession is cosmological (debated for some time), these were at great distances and highly luminous. Two notable features:
1. Flux is seen bluewards of Ly$\alpha$ emission peak.
2. Occasionally, absorption lines are seen along the LOS.

Led to two influential back-to-back papers in 1965 ApJ, by Gunn & Peterson, and Bahcall & Salpeter. Gunn and Peterson argued that it’s remarkable that any flux is seen at all, because if cosmos was neutral then HI absorption should strongly absorb that flux; hence diffuse gas must be highly ionized.

Bahcall & Salpeter argued that absorption lines were actually redshifted Ly$\alpha$ along the line of sight (rather than some unkown element associated with the quasar). The picture developed that the IGM was highly ionized but dotted by pockets of neutral gas (possibly associated with ISM of other galaxies).

Gunn & Peterson’s result still gives valuable insight today, while Bahcall & Salpeter’s turned out not to be the right picture; this wasn’t fully realized until the 90’s.

Gunn & Peterson’s argument was that a smooth neutral IGM would have enormous optical depth below 1216Å (HI Ly$\alpha$, 2-1 transition). That we observe flux bluer than this means that the IGM is almost entirely ionized. For a number density of neutral hydrogen atoms $n(z)$, with cross-section $\sigma(\nu(1+z))$, we get an optical depth of

$$\tau = \int_{0}^{z_{0}} n(z)\sigma(\nu(1+z))(dl/dz)dz$$

Now $n(z) = n_{HI}(1+z)^3$ for a constant comoving number density of HI atoms. $dl/dz = c/H(z) = cH_{0}^{-1}(1+z)^{-3/2}$ for a EdS cosmology. $\sigma = \pi e^2 f/m_e c$ (where $f$ is oscillator strength=0.416 for Ly$\alpha$) and is highly peaked at the (redshifted) resonance of HI, namely 1216Å.

$$\tau(z) \approx n_{HI}\sigma(\nu_{0}(1+z))cH_{0}^{-1}(1+z)^{3/2}$$
Plugging in some numbers and noting that $n_H \approx \rho_c \Omega_b / m_p$ gives

$$\tau = 6.4 \times 10^5 h^{-1} \left( \frac{\Omega_b h^2}{0.02} \right) \left( \frac{1 + z}{3} \right)^{3/2} \left( \frac{n_{HI}}{n_H} \right)$$

Schmidt observed 3C9 to be at $z = 2.01$. The mean optical depth was $\sim 0.5$. For $H_0 = 10^{10}$ yr$^{-1}$, $\Omega_m = 1$, Gunn & Peterson calculated that $\Omega_{HI} \approx 2 \times 10^{-7}$. For WMAP parameters, this gives a neutral fraction of $3.8 \times 10^{-6}$. This turns out to be remarkably close, given the simplicity of the argument.

There must be a sea of UV photons (above 13.6 eV) in the universe to keep the IGM ionized. So: universe becomes neutral at $z = 1000$, else the optical depth to $z = 1000$ would be far larger than we observe (they’ve done this calculation in homework). However, we observe the IGM to be ionized, certainly out to $z = 6$ and probably beyond. Hence, the IGM must be reionized at some redshift between 6 and about 50.

Figure, SDSS quasar spectra, discovery of G-P absorption trough

## 2 Basic Models of Reionization

The baryonic pre-galactic medium (PGM) evolves in three distinct phases. At high redshifts ($z > 1100$) the PGM is hot, fully ionized, and optically thick to Thomson scattering, and hence coupled to the photon field. As the universe expands, the PGM cools, and eventually recombines, leaving a surface of last scattering (the cosmic microwave background, CMB), plus a neutral PGM. This neutral phase lasts from $z = 1100$ to $z \sim 14$. At some point between $z \sim 14$ and 6, hydrogen in the PGM is ‘reionized’, due to UV radiation from the first luminous objects, leaving the fully reionized intergalactic medium (IGM) seen during the ‘realm of the galaxies’ ($6 > z > 0$). The ionized, dense PGM at very high redshift has been well studied through extensive observations of the CMB. Likewise, the reionized, rarified IGM at low redshift has been well characterized through QSO absorption line studies. The middle phase – the existence of a neutral IGM during the so-called ‘dark ages’ (Rees 1998), and the process of reionization of this medium, is the last directly observable phase of cosmic evolution that remains to be verified and explored. The epoch of reionization (EoR) is crucial in cosmic structure formation studies, since it sets a fundamental benchmark indicating the formation of the first luminous objects, either star forming galaxies or active galactic nuclei (AGN).

Assuming reionization is driven by UV photons from the first luminous sources (stars or AGN),
analytic and numerical, calculations suggest that reionization follows a number of phases. The first phase will be relatively slow, with each UV source isolated to its own Stromgren sphere. Eventually, these spheres grow and join, leading to much faster reionization, due to the combination of accelerating galaxy formation, plus the much larger mean free path of ionizing photons in the now porous IGM. This stage is known as ‘overlap’, or ‘percolation’. The last phase entails the etching-away of the final dense filaments in the IGM by the intergalactic UV radiation field, leading to a fully ionized IGM (neutral fraction, \(x_{HI} \sim 10^{-5}\), at \(z = 0\)).

Figure: overlap picture from Loeb and Haiman.

Some of the key questions regarding reionization include:

- **When** did reionization happen? Whether it is early (\(z \sim 15\)) or late (\(z \sim 6 - 8\)), whether it has an extended history or resembles a phase transition.

- **How** did reionization proceed? Whether it is homogeneous or with large scatter? How did HII regions grow during reionization and how did overlap happen?

- **What** did it? Whether the main sources of reionization photons come from dwarf galaxies, AGNs, or even more exotic sources such as decay particles.

### 3 Constraint on the end of reionization

Figure, deepest troughs

Figure, \(\tau\) evolution

Figure, neutral fraction evolution.

Fan et al. measured the evolution of Gunn-Peterson optical depths along the line of sight of the nineteen \(z > 5.7\) quasars from the SDSS. We found that at \(z_{abs} < 5.5\), the optical depth can be best fit as \(\tau \propto (1 + z)^{4.3}\), while at \(z_{abs} > 5.5\), the evolution of optical depth accelerates: \(\tau \propto (1 + z)^{10}\). There is also a rapid increase in the variation of optical depth along different lines of sight: \(\sigma(\tau)/\tau\) increases from \(\sim 15\%\) at \(z \sim 5\), to \(> 30\%\) at \(z > 6\), in which \(\tau\) is averaged over a scale of \(\sim 60\) comoving Mpc. Assuming photoionization equilibrium and a model of IGM density distribution, one can convert the measured effective optical depth to IGM properties, such as the level of UV ionizing background and average neutral fraction. We find that at \(z > 6\) the volume-averaged neutral fraction of the IGM has increased to \(> 10^{-3.5}\), with both ionizing background and neutral fraction experiencing about one order
of magnitude change over a narrow redshift range, and the mean-free-path of UV photons is shown to be $< 1$ physical Mpc at $z > 6$. However, with the emergence of complete Gunn-Peterson troughs at $z > 6$, it becomes increasingly difficult to place stringent limits on the optical depth and neutral fraction of the IGM.

Briefly, flavors of these include:

- Quasar stromgren sphere
- IGM damping wing
- Ly$\alpha$ galaxies

## 4 Other Observational Constraints

### 4.1 CMB

Thomson scattering produces CMB polarization when free-electron scatterers are illuminated by an anisotropic photon distribution. Large scale CMB polarization therefore probes the ionization history by measuring the total optical depths to be CMB produced by free electrons generated from reionization process. The history of CMB polarization measurement is a rather complicated and confusing one. The first WMAP result came as a surprise, with $\tau \sim 17$, implying a very high reionization redshift, $z \sim 17$, assuming reionization is a phase transition in hydrogen ionization state. However, the subsequent WMAP CMB $\tau$ become progressively smaller, with Planck results at even lower value. This is indeed a rather confusing episode in the reionization field. The initial quasar results strongly suggest reionization end at $z \sim 6$. But first WMAP result suggests a much higher redshift. This would imply a prolonged reionization history, even a double reionization model.

Figure: history of $\tau$ measurement. Now the best guess is $z = 7 - 9$, consistent with quasar observations.

However, the CMB polarization only measures an integrated reionization signal. Figure illustrates the degeneracy of reionization redshift and detailed reionization history based on CMB data alone.
4.2 21 cm Probe

The 21cm line of neutral hydrogen presents a unique probe of the evolution of the neutral intergalactic medium, and cosmic reionization. Furlanetto & Briggs (2004) point out some of the advantages of using the HI line in this regard: (i) unlike Ly$\alpha$ (ie. the Gunn-Peterson effect), the 21cm line does not saturate, and the IGM remains ‘translucent’ at large neutral fractions. And (ii) unlike CMB polarization studies, the HI line provides full three dimensional (3D) information on the evolution of cosmic structure, and the technique involves imaging the neutral IGM directly, and hence can easily distinguish between different reionization models. HI 21cm observations can be used to study the evolution of cosmic structure from the linear regime at high redshift (ie. density-only evolution), through the non-linear, ‘messy astrophysics’ regime associated with luminous source formation. As such, HI measurements are sensitive to structures ranging from very large scales down to the source scale set by the cosmological Jeans mass, thereby making 21cm the “richest of all cosmological data sets”

In the Raleigh-Jeans limit, the observed brightness temperature (relative to the CMB) due to the HI 21cm line at a frequency $\nu = \nu_{21}/(1 + z)$, is given by:

$$T_B \approx \frac{T_S - T_{CMB}}{1 + z} \tau \approx 7(1 + \delta)x_{HI}(1 - \frac{T_{CMB}}{T_S})(1 + z)^{1/2} \text{mK}, \quad (1)$$

The conversion factor from brightness temperature to specific intensity, $I_\nu$, is given by:

$$I_\nu = \frac{2k_B}{(\lambda_{21}(1+z))^2}T_B = 22(1 + z)^{-2}T_B \text{Jy deg}^{-2}.$$  

This shows that for $T_S \sim T_{CMB}$ one expects no 21cm signal. When $T_S >> T_{CMB}$, the brightness temperature becomes independent of spin temperature. When $T_S << T_{CMB}$, we expect a strong negative (ie. absorption) signal against the CMB.

The interplay between the CMB temperature, the kinetic temperature, and the spin temperature, coupled with radiative transfer, lead to a number of interesting physical regimes for the HI 21cm signal: (I) At $z > 200$ equilibrium between $T_{CMB}$, $T_K$, and $T_S$ is maintained by Thomson scattering off residual free electrons and gas collisions. In this case $T_S = T_{CMB}$ and there is no 21cm signal. (II) At $z \sim 30$ to 200, the gas cools adiabatically, with temperature falling as $(1 + z)^2$, ie. faster than the $(1+z)$ for the CMB. However, the mean density is still high enough to couple $T_S$ and $T_K$, and the HI 21cm signal would be seen in absorption against the CMB (Sethi 2005). (III) At $z \sim 30$, collisions can no longer couple $T_K$ to $T_S$, and $T_S$ again approaches $T_{CMB}$. However, the Ly$\alpha$ photons from the first luminous objects (Pop III stars or mini-quasars), may induce local coupling of $T_K$ and $T_S$, thereby leading to some 21cm absorption regions. At the same time, Xrays from these same objects could lead to local IGM warming above $T_{CMB}$. Hence one might expect a patch-work of regions with
no signal, absorption, and perhaps emission, in the 21cm line. (IV) At low-z, all the physical processes come to play. The IGM is being warmed by hard Xrays from the first galaxies and black holes, as well as by weak shocks associated with structure formation, such that $T_K$ is likely larger than $T_{CMB}$ globally. Likewise, these objects are reionizing the universe, leading to a fundamental topological change in the IGM, from the linear evolution of large scale structure, to a bubble dominated era of HII regions.

Difficulties: small signal, strong foreground, radio frequency interference, ionosphere variation.

Needs: large area, quite site, model astrophysical foreground.

Many programs have been initiated to study the HI 21cm signal from cosmic reionization, and beyond. The largest near-term efforts are the Mileura Wide Field Array (MWA$^1$), the Primeval Structure Telescope (PAST$^2$), and the Low Frequency Array (LOFAR$^3$). These telescopes are being optimized to study the power spectrum of the HI 21cm fluctuations. In the long term the Square Kilometer Array (SKA$^4$) should have the sensitivity to perform true three dimensional imaging of the neutral IGM in the 21cm line during reionization. And at the lowest frequencies ($< 80$ MHz), the Long Wavelength Array (LWA$^5$), and eventually the Lunar array (LUDAR), are being designed for the higher $z$ HI 21cm signal from the PGM.

5 Source of Reionzation

Regardless of the detailed reionization history, the IGM has been almost fully ionized since at least $z \sim 6$. This places a minimum requirement on the emissivity of UV ionizing photons per unit comoving volume required to keep up with recombination and maintain reionization:

$$\dot{N}_{ion}(z) = 10^{51.2} \text{s}^{-1} \text{Mpc}^{-3} \left( \frac{C}{30} \right) \times \left( \frac{1 + z}{6} \right)^3 \left( \frac{\Omega_b h^2}{0.02} \right)^2,$$

where $C \equiv \langle n_e^2 / \langle n_i^2 \rangle \rangle^2$ is the clumping factor of the IGM. It is difficult to determine $C$ from observations, and has large uncertainty when estimated from simulations ($C = 10 - 100$). At $z < 2.5$, the UV ionizing background is dominated by quasars and AGN. At $z > 3$, the density of luminous quasars decreases faster than that of star-forming galaxies, and the ionizing background has an increased contribution from stars.

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1. web.haystack.mit.edu/arrays/MWA/LFD/index.html
2. web.phys.cmu.edu/past/
3. www.lofar.org/
5. lwa.unm.edu/index.shtml
Quasars and AGN are effective emitters of UV photons. Luminous quasar density declines exponentially towards high-redshift: it is \(\sim 40\) times lower at \(z \sim 6\) than at its peak at \(z \sim 2.5\). However, quasars have a steep luminosity function at the bright end – most of the UV photons come from the faint quasars that are currently below the detection limit at high-redshift. Based on the derived luminosity function, we find that the quasar/AGN population cannot provide enough photons to ionize the intergalactic medium (IGM) at \(z \sim 6\) unless the IGM is very homogeneous and the luminosity at which the QLF power law breaks is very low.

Due to the rapid decline in the AGN populations at very high \(z\), most theoretical models assume stellar sources reionized the universe. Figure shows the present constraints on the evolution of UV luminosity and star formation rate of Lyman Break galaxies. Comparing to AGNs, galaxy population show only mild evolution at \(z > 5\). However, despite rapid progress, there is still considerable uncertainty in estimating the total UV photon emissivity of star-forming galaxies at high-redshift, especially the IMF and the UV escape fraction from dwarf galaxies. Given these uncertainties, the current data are consistent with star forming galaxies, in particular, relatively low luminosity galaxies, as being the dominant sources of reionizing photons, although more exotic sources, such as high-redshift mini-quasars, can not yet be ruled out as minor contributors of reionization budget. Issue with escape fraction.

A totally complementary probe provided by redshifted 21cm emission and absorption. Since lots of neutral gas, should have lots of 21cm hyperfine emission. Depends on spin temperature \((T_s)\) vs. gas temperature set by CMB \((T_{CMB} \propto (1 + z)^{-1})\). Brightness temperature

\[
\delta T_b \propto (1 - T_{CMB}/T_s)
\]  

Because of adiabatic expansion, gas cools faster than CMB initially \((P \propto \rho T, \text{ plus EOS } P \propto \rho^{5/3} \text{ gives } T_s \propto (1 + z)^2)\). But then, gas becomes too diffuse to keep 21cm in equil, and CMB heating raises \(T_s\) again. When photons turn on \((z < \sim 30), T_s > T_{CMB}\), and 21cm can map cosmic web! Near-term radio arrays won’t have sensitivity/resolution to see anything but vague blobs. But SKA should. Huge problems with foregrounds not clearly solved yet.

6 Helium reionization

In simple models of ionizing background evolution, stars dominate the early growth of the background, then quasars become progressively more important at \(z < 4\). Since quasar light is much harder, it can ionize HeII into HeIII (which needs 4 Ry photons, i.e. 54.4 eV). This process is known as Helium reionization, and has several consequences:
1. The global ionizing background becomes harder. This is important for metal species such as OVI whose ionization potential is $\sim 9$ Ry.

2. Latent heat is released, causing energy injection into the IGM. If this occurs on timescales faster than Hubble expansion can equilibrate, then the IGM temperature may spike. Some claims to see this at $z \sim 3.2$, but controversial.

3. Offers an opportunity to study reionization at an observationally accessible redshift ($z \sim 3$), perhaps giving insights to how H reionization proceeds at $z > 6$. Downside is that photons are 304Årest, so need far UV spectroscopy to see at $z \sim 3$. Major goal of COS on HST.

7 Lyα Forest

Lyα cloud models

An alternative hypothesis was that Lyα absorbers were ejected from the quasar. Energetically feasible since quasars were highly luminous. Indeed, today ejected absorption is seen up to 10’s of thousands of km/s away. But Sargent et al. (1980) showed that Lyα forest was not ejected but intervening, by mainly showing that Lyα absorbers were distributed uniformly along LOS (while an ejection model would predict many more at low velocities), and there was no correlation between line strength and (presumed) ejection velocity.

Sargent et al also showed that Lyα lines were uncorrelated, unlike what would be expected if they arose in galaxies along LOS. Hence they proposed cosmologically distributed cold clouds, pressure-confined by a warm IGM ($n \sim 10^{-5}, T \sim 10^5 K$). By measuring $N_{HI}$ and assuming sphericity, estimated mass density in such cold clouds to be quite small, $\Omega_c \sim 10^{-3}$. This turned out to be wrong, but initiated a bevy of Lyα absorber models.

The main competing idea in the 80’s was that Lyα clouds were “minihalos”, i.e. gravitationally-confined pockets of gas. As data improved, both confinement models needed fine-tuning to what eventually became rather elaborate levels, with non-spherical geometries and no obvious source of such pockets.

Another difficulty with confinement models is that nearby LOS showed correlated Lyα absorption features over 0.5 – 1 Mpc away. Hence Lyα absorbers are large!

Advent of high-resolution quasar spectroscopy put to death such confinement models, while advancing numerical simulations offered a new and elegant structure-formation based interpretation. We will discuss observations today, modern Lyα models next time.

Observations of the Lyα forest
Echelle spectroscopy (dispersing light into orders on a CCD) on large telescopes provided first fully resolved spectra of Lyα forest in distant quasars. In problem set you will compute that the thermal linewidth of Lyα absorbers is \( \sim 10 \text{ km/s} \), hence need this spectral resolution in order to resolve every absorber. Keck’s HIRES first to provide a detailed view; now VLT’s UVES does even better.

High-z quasar (Figure).

Quasar’s Lyα emission peak at \( \lambda = (1 + z) \times 1216\text{Å} \),
Bluewards see Lyα forest absorption due to intervening HI.
Bluewards of \( \lambda = (1 + z) \times 1025\text{Å} \) we get Ly-β forest superimposed.
Note decline in flux bluewards: Intrinsic quasar power law + falling detector efficiency.
See truncation at \( \lambda \approx 4000\text{Å} \) due to LLS at \( z \approx 3.4 \).
Note absorption redwards of Lyα: Metals!

Taxonomy of Lyman alpha absorbers and the connection to \( N_{\text{HI}} \):
1. \( N_{\text{HI}} < 10^{14} \text{ cm}^{-2} \): no saturation
2. \( 10^{14} < N_{\text{HI}} < 10^{17} \): saturation depending on line width in velocity space
3. \( N_{\text{HI}} > 10^{17} \): Lyman limit system (LLS)
4. \( N_{\text{HI}} > 2 \times 10^{20} \): Damped Lyman alpha (DLA) system

Explain Voigt profile and curve of growth.

DLAs are so named because at those \( N_{\text{HI}} \) the Lorentzian damping wing of the Voigt profile becomes prominent. In fact, the division between DLA & LLS is purely historical, as modern spectrographs can detect damping wings down to \( N_{\text{HI}} \sim 10^{19} \) (now called “sub-DLAs”).
Looking thru ISM of MW would give \( N_{\text{H}} \gg 10^{21} \text{ cm}^{-2} \), i.e. a DLA.

LLS are so named because at \( N_{\text{HI}} > 10^{17} \) gas is optically thick at the Lyman limit, i.e. all incident ionizing photons are absorbed; this is a \textit{physical} definition. Below this is known as Lyα forest, although “partial” LLS at \( N_{\text{HI}} > 10^{16} \) are still generally thought to be associated with galaxies’ ISMs (e.g. seen to contain low-ionization metals).

Metal lines and BALs.

Characterizing Lyα forest absorbers

Each absorber can be fit with three parameters: redshift, column density \( N_{\text{HI}} \), and line width (called \( b \)-parameter). High-resolution spectra are fit with a large number of superposition of Voigt profiles. Note for \( N_{\text{HI}} \ll 10^{20} \) the Voigt profile is just a Gaussian, as the Lorentzian damping wing is unimportant.
The most basic counting statistic is column density distribution: Number of lines per unit redshift interval per unit log column density. Observed to be a power law $f(N_{\text{HI}}) \propto N_{\text{HI}}^{-\beta}$, with $\beta \sim 1.5$ at $z \sim 3$.

Aside: Note that Ly$\alpha$ forest is much better characterized at $z > 2$ where Ly$\alpha$ falls into optical, than at $z < 2$ where space-based UV observations are required!

The statistic associated with $b$-parameter is just a histogram called the linewidth distribution.

At $z \sim 3$, this peaks at $\sim 25$ km/s, with a roughly truncated Gaussian to smaller $b$ and a long tail to large $b$.

Generally this is understood in terms of a thermal component and a bulk flow component:

$$b^2 = \frac{2kT}{m_p} + b_{\text{bulk}}^2 \quad (4)$$

As we will see next time, $b_{\text{bulk}}$ arises because Ly$\alpha$ clouds are large and still expanding with Hubble flow, causing an intrinsic redshift spread within the cloud.

Minimum $b$ (i.e. the low-$b$ truncation) corresponds to pure thermal broadening. Measuring this is complicated by the fact that lines are heavily blended and not necessarily pure Gaussian (as we will see next time), hence small $b$ lines can be wings of a larger line rather than a true low-$T$ absorber. Nevertheless, it is possible to estimate $b_{\text{thermal}} \approx 1.5 - 2 \times 10^4$K (Schaye et al 2000).

The statistic associated with redshift is line-of-sight clustering. High-quality data confirmed Sargent et al’s result that Ly$\alpha$ lines have low clustering, much less than e.g. galaxies. In detail, clustering increases with $N_{\text{HI}}$, with a slight signature first appearing at $N_{\text{HI}} \sim 10^{14}$.

Redshift evolution

Ly$\alpha$ forest undergoes a transition at $z \sim 1 - 1.5$, from rapidly-evolving to slowly-evolving. Parameterized as $dN/dz = (1 + z)^{\gamma}$, where $\gamma \approx 2.8$ at $z > 1.5$, and $\gamma \approx 0.5$ at $z < 1.5$. Initially postulated as two absorber populations.

Initially postulated as two absorber populations. Absorbers expanding with Hubble follow, or are self-gravitating clouds. Now let’s consider a static population of small neutral clouds, with fixed comoving number density $\phi_0$ and comoving radius $r_0(1+z)$ (i.e. constant physical radius $r_0$). Then in a path length $\Delta z$ one expects to find the following number of absorbers:

$$\frac{dN}{dz} \Delta z = \pi r_0^2 \phi_0 (1 + z)^2 \frac{dS}{dz} \Delta z, \quad (5)$$

where $S(z) = \int c dz/H(z)$ is the comoving path length. So

$$\frac{dN}{dz} \propto (1 + z)^2 \frac{dS}{dz} \propto \frac{(1 + z)^2}{H(z)} \quad (6)$$
Putting in a rough value of $H(z) \propto (1 + z)^{3/2}$ (for $\Omega = 1$), we get $\gamma = 0.5$. This agrees with low-$z$ evolution.

Hence the idea was that high-$z$ absorbers were dominated by large, uncollapsed clouds while low-$z$ were essentially galaxy/halo gas (whose size and comoving number density evolve slowly since $z = 1$). It turns out this interpretation, like confinement models, was shown to be incorrect in the “new” model for the Ly$\alpha$ forest. While the low-$z$ scaling was correct in the cloud model, the amplitude for realistic sizes of gaseous halos was too small; basically, it is very rare for a random LOS to pass thru a galactic halo.

Instead the change in evolution reflects mostly an evolution in the photoionization rate $\Gamma_{\text{HI}}$, which drops rapidly since $z \sim 2$ owing to the rapid decline in the quasar number density.

**FGPA**

We have Gunn-Peterson optical depth:

$$\tau = 6.4 \times 10^5 h^{-1} \left( \frac{\Omega_b h^2}{0.02} \right) \left( \frac{1 + z}{3} \right)^{3/2} \left( \frac{n_{\text{HI}}}{n_{\text{H}}} \right).$$

Changing the density alters the ionization balance and the amount of neutral Hydrogen. Hence, density fluctuations produce variations in the optical depth and produce the Lyman alpha forest. Ignoring peculiar velocities and thermal broadening, then the optical depth depends only on the underlying baryon density. Because pressure forces as small, the baryon density traces the dark matter density.

Thus the Ly$\alpha$ forest represents a 1-D probe of the spectrum of mass fluctuations. Ly$\alpha$ lines should not be thought of as individual features, but as a continuous flux distribution. This is the Fluctuating Gunn Peterson Approximation.

The Ly$\alpha$ forest arises from absorption of Ly$\alpha$ photons by neutral hydrogen gas in the intergalactic medium (eq. 2). Assuming local photoionization-recombination equilibrium, we have:

$$n_{\text{HI}} \Gamma = n_{\text{HII}} n_e \alpha(T),$$  (7)

where $n_{\text{HI}}$, $n_{\text{HII}}$ and $n_e$ are the *local* densities of neutral, ionized hydrogen and electrons in the IGM, respectively, $\Gamma$ is the photoionization rate, and $\alpha(T)$ is the recombination coefficient at temperature $T$ (Abel et al. 1997),

$$\alpha(T) = 4.2 \times 10^{-13} (T/10^4K)^{-0.7} \text{cm}^3\text{s}^{-1}.$$  (8)
The photoionization rate $\Gamma$ is related to the ionizing background flux $J_\nu$ by,

$$\Gamma = 4\pi \int \frac{J_\nu}{h\nu} \sigma_\nu d\nu,$$

where the integral includes the ionizing photons from the HI Lyman Limit to the HeII Lyman Limit assuming that the He in the IGM is singly ionized, and $\sigma_\nu$ is the HI cross section of ionizing photons, $\sigma_\nu \sim \nu^{-3}$.

If the IGM is mostly ionized by a uniform ionizing background, the evolution of the optical depth can be expressed as (Weinberg et al. 1997),

$$\tau_{\text{GP}} \propto (1 + z)^6 (\Omega_b h^2)^2 \alpha(T) \propto \frac{(1 + z)^{4.5}(\Omega_b h^2)^2 \alpha(T)}{h\Gamma \Omega_m^{0.5}}.$$

The Ly$\alpha$ absorption increases rapidly with increasing redshift even if the ionizing background remains constant with redshift.

$$\tau \propto n_{\text{HI}} \propto n_{\text{HI}}^2 T^{-0.7}/\Gamma_{\text{HI}}$$

And the combination of adiabatic cooling and photoionization heating in an expanding universe gives a relation:

$$T \propto \rho^{0.6}.$$

Use FGPA to measure:

- **Matter power spectrum** on intermediate scales at high redshifts. Since $\rho_{\text{gas}} \propto \rho_{\text{dark}} \propto \tau_{\text{HI}}^{1.6}$, we can compute fluctuations in $\rho$ from fluctuations in $\tau_{\text{HI}}$. Obtaining $\tau_{\text{HI}}$ from line-of-sight fluxes is complicated by saturation; fortunately, at $z \sim 2 - 4$ saturated lines comprise a tiny portion of the spectrum.

Scales: Can’t probe too close to thermal broadening, since assumption of gas-follows-DM breaks down in velocity space. So hard limit $< 10$ km/s (30 kpc comoving at $z = 3$). For precision work, limit is much larger, so this limits to $> several$ hundred kpc.

To obtain large samples, need faint targets hence low-res spectra. For ex SDSS spectra good enough for this (cf. BOSS), with resolution of $\sim 200$ km/s, i.e. $\sim 1$ Mpc.

On large scales, no limit in principle, but one must know that $\Gamma_{\text{HI}}$ is spatially uniform (or know its variations). Since QSOs dominate, it is probably non-uniform on clustering scale of QSOs, which is tens of Mpc.

So FGPA effective on scales of $\sim 1 - 10$ Mpc. Also probes an interesting $z$ range, between LSS ($z < 1$) and CMB ($z = 1088$), and generally smaller scales. Hence Ly$\alpha$
forest, along with WMAP and LSS (e.g. BAO, clusters, or WL), provides the three pillars for constraining cosmological parameters and (eventually) dark energy.

- **Baryon density** \( (\Omega_b h^2 \approx 0.02) \). Since \( \bar{\tau}_{HI} \propto \rho_{gas}^{1.6} \Gamma_{HI}^{-1} \), by measuring mean optical depth and estimating photoionization rate it is possible to get mean baryon density, i.e. \( \Omega_b \). Estimate by Rauch et al (1997) found \( \Omega_b > 0.017 h^{-2} \), consistent with WMAP and BBN. BBNS ([D/H]) can constrain \( \Omega_b \), but in early 90’s gave conflicting values. Deuterium lies 82 km/s bluewards of Ly\( \alpha \), but is down by 5 orders of mag in \( \tau \). So must find strong but narrow line. In late 90’s, Burles & Tytler had series of papers giving \( \Omega_b \sim 0.02 h^{-2} \), confirming Ly\( \alpha \) forest results. Then WMAP clinched it. These days, argument can be turned around to measure \( \Gamma_{HI} \) given \( \Omega_b \); in fact, this gives the most precise measure of \( \Gamma_{HI} \) at both high and low redshift!

- **Metallicity of IGM.** Heavy elements are observed in the diffuse IGM as metal absorbers, e.g. CIV. From this can measure ratios like CIV/HI. What is desired is metallicity, e.g. [C/H]. Need ionization corrections. Using \( \tau_{HI} I \), one can estimate \( \rho_{gas} \) and \( T \) using FGPA, which then give the ionization corrections for C and H, allowing one to correct CIV/HI into C/H.

8 Baryon census

Baryons seen in emission make up a small fraction of BBN or CMB inferred total (\( \Omega_b \approx 0.045 \)). Most baryons today are not observed to be in stars or cold, dense gas in galaxies: Integrated LF gives 8-10% stars; 21cm/CO emission gives 1% in cold, dense gas; X-ray emission from clusters gives 10% in hot ICM. Where is the rest? Intergalactic!

Universe begins with most baryons in “intergalactic medium”, i.e. gas that has not (yet) collapsed into galaxies. Physical properties:
1. Gas pressure forces are low, hence gas mostly traces dark matter.
2. Gas densities are low, so radiative cooling is unimportant.
3. Optically thin, so radiation mostly passes thru.
4. Highly ionized owing partly to background radiation from various sources.

IGM comes in three basic forms:
1. Cool photoionized intergalactic gas tracing uncollapsed LSS (\( T \sim 10^4 \)K), detectable as HI absorption to background sources.
2. “Warm-hot” intergalactic medium (WHIM); gas shock heated by collapse on LSS to
$T \sim 10^{1.5} - 10^{6.5} \text{K}$, which is too hot to contain HI and too cold to emit in X-rays. Sometimes called the “missing baryons”.

3. Intracluster medium (ICM); gas shock heated by virialization within large halos to $T \sim 10^7 - 10^8 \text{K}$, emitting in X-rays.

Simulations indicate currently roughly equally divided between “bound” baryons (including ICM), Ly$\alpha$ forest, and “warm-hot” gas. But in the past, most in Ly$\alpha$ forest (Figure).