Assignment 3
Astronomy 541

Back-of-the-envelop calculations (not graded, no need to turn in)

1) Could the energy in the cosmic microwave background have been produced by stars? (Use metallicity as a constraint.)

2) The WMAP team reports: $\Omega_b = 0.045$, $\Omega_m = 0.28$, $h = 0.7$, and an optical depth of 0.09 to Thomson scattering. At what redshift was the universe reionized?
Assignment: Due Thursday Oct 13, in class

**Problem 1 (5 pts):** For a universe composed only of radiation ($w = 1/3$), compute the time $t(z)$ between the Big Bang and a given redshift (note that this formula will be slightly different than the formula in Assignment 2). Express this time both in terms of $H_0$ and redshift and in terms of the Hubble parameter at the final redshift $H(z)$.

Compute the comoving distance $r_H(z)$ travelled by a (non-interacting) light ray between the Big Bang and a given redshift (again, note that this formula is slightly different than the $r(z)$ formula you’ve been using). Express this in terms of $z$ and $t(z)$ and interpret the result.

**Problem 2 (5 pts):**

a) Compute $r_H(z)$ for a general cosmology of radiation, matter, curvature, and a cosmological constant. However, you should confine yourself to early times (high redshift) where the contribution to $H(z)$ from curvature and a cosmological constant is negligible. In other words, assume $H(z) = H_0 [\Omega_r (1 + z)^4 + \Omega_m (1 + z)^3]^{1/2}$ but allow $\Omega_r + \Omega_m \neq 1$. Note: this integral should be done analytically, not numerically.

Demonstrate that the limits of $r_H(z)$ at early times match the behavior of Problem 1.

b) The CMB plus the predicted neutrino backgrounds make $\Omega_r h^2 = 4.2 \times 10^{-5}$. For a universe with $h = 0.7$ and $\Omega_m = 0.3$, compute the value of the $r_H$ at $z = 1000$. This is the comoving distance that a causal signal can propagate prior to $z = 1000$ in a universe of radiation and matter. How does this size compare to the present-day size of the universe?

Repeat the calculation for the $z = z_{eq}$, where $z_{eq}$ is the redshift at which the matter and radiation energy densities are equal.

These are examples of a particle horizon. Extrapolating to early times, the causally connected portion of the universe is shrinking to zero in comoving coordinates! This raises the question of how we can appeal to early causal physics to explain the homogeneity of the universe.

**Problem 3 (5 pts):**

a) If the atoms in the universe were ionized, then the free electrons would scatter photons (at least non-gamma-rays) at the Thompson cross-section $\sigma_T = 6.65 \times 10^{-25}$ cm$^2$. Assume that the density of nucleons is $3.8 \times 10^{-31}$ g cm$^{-3}$ today ($\Omega_b h^2 = 0.02$) and consider that all of the baryons are in hydrogen (ignore helium). Charge neutrality insists that the number of electrons is equal to the number of protons. If the universe were fully ionized, compute the optical depth a photon encounters on its way from redshift $z$ to us. Assume that the universe has $\Omega_0 = 1$ and ignore any radiation contribution to the Hubble constant.

Hints: Compute the density of electrons today and write down the scaling with redshift. The optical depth is then the integral along the line of sight of the cross-section times the density. You can change variables from $d\ell$ to $dz$. It sounds more complicated than it is! See also pages 280 of Longair.

b) At what redshift is the optical depth unity? Assume $H_0 = 70$ km s$^{-1}$ Mpc$^{-1}$ (i.e. $h = 0.7$).

**Problem 4 (5 pt):**

Consider that we change particle physics so as to include a yet-undiscovered stable massive particle. For simplicity, we will make it spin-0 (meaning that $g = 1$) and call it $X$. $X$ and
its antiparticle $\bar{X}$ interact quickly enough in the early universe that their number densities reach thermal equilibrium. We will imagine that the mass $m_X$ is large, of order the proton mass or larger.

If $X$ interacts rarely enough, then it will decouple (interaction rate less than Hubble parameter) when the universe is still hotter than $m_X c^2$. In that case, $X$ remains in a thermal distribution today. Show that this is a cosmological catastrophe by computing $\Omega_m h^2$ in terms of $m_X$ (where the latter is measured in GeV).