Final Review
Astronomy 541

1 Basic Format

Final exam will be Tuesday Dec 13 at 3:30pm. You can either do it in room N305 or bring it back to your office to take it. It is 120 min. This exam is closed book and closed notes. You may use a calculator. I will provide a formula sheet with the exam which shall include the equations and numerical constants that you will need for the exam. However, there are some definition and scaling relation that you should know by heart (e.g., Friedmann equation, virial theorem, Einstein radius). I have included a superset of those equations that we encountered during our class. A good starting point for your review is to understand their meaning. You do not need to memorize the more difficult ones, or how to derive them in detail, but you should know the physical meanings and applications.

There are 11 questions, worth a total of 100 points. The first eight questions require very short answers and no derivations; the next question asks you to discussion in one or two short paragraphs about a concept; the last two questions require you to do some derivations. Examples:

Problem 1 (5 pts): Define galaxy power spectrum and its relation with galaxy two-point correlation function.

Problem 2 (10 pts): Describe thermal S-Z effect and its frequency dependence; how to use S-Z effect, combined with X-ray observations, to measure the density and temperature of a cluster and to measure Hubble constant; discuss the main assumptions made in these measurements.

Example quantitative question will be one of the shorter (5 point) homework questions, or half of the long homework questions.

The first two kinds of questions are very similar to those you will encounter in your close-booked prelim; the quantitative one will be similar in complexity for the open-booked prelim questions, although those in prelim tend to be somewhat broader in the area of knowledge they are testing while those in your final only concerns cosmology.
2 Basic Concepts to Review

- **Classical Cosmology**: Importance of homogeneity, isotropy, and expansion of the Universe as the basis for the cosmological model; Metric, and the FRW metric; cosmological redshift; Hubble expansion from metric; cosmological distances, volume and time; horizons; Friedmann equations; Recasting of Friedmann eq as $H(z)$; Interplay of density, curvature, and destiny of the Universe.

- **Cosmological Tests**: Trends of distance, volume, and time for different cosmologies; classical cosmological tests - $H_0$ and density parameters; Distance ladders, direct distance methods, SNe, etc.; Age of the Universe; dark energy: evidence, parameterization

- **Hot Big Bang**: Cosmic microwave background – basic implications; Recombination; Thermal history of the Universe; Big Bang nucleosynthesis – basic pathways, Helium-4, D/H, observations

- **Perturbations and Large Scale Structure**: linear perturbation theory; solution for growth functions; Jeans instability; correlation function, power spectrum; Harrison-Zeldovich power spectrum; $\sigma$ and $\sigma_8$; physics of CDM power spectrum; redshift survey

- **CMB**: Basic physics; Sachs-wolfe effect; Acoustic peaks; Silk damping; What we learn about cosmology from the CMB power spectrum, e.g., parameter dependences; polarization and reionization.

- **Spherical Collapse**: Zel’dovich approximation; Spherical collapse; Virialization and halo scalings; NFW profiles; Press-Schechter and halo masses;

- **First galaxies**: Cooling in galaxies, mass scale, IMF, observations of high-z galaxies, Lyman break galaxy

- **Reionization and IGM**: basic physical processes, observational constraints, Gunn-Peterson effect; Lyman $\alpha$ absorbers; the fluctuating Gunn-Peterson approximation

- **Galaxy Evolution**: The hierarchical galaxy formation cartoon; problems in galaxy formation

- **Gravitational lensing**: lens equation for a point mass; microlensing; weak lensing; singular isothermal sphere

- **Clusters**: Basic properties Mass estimation from X-ray gas, lensing, S-Z, velocity dispersion; clusters and cosmology
Formula Sheet

The following formulae could be of use on the exam. You are responsible for knowing what the symbols mean and when the formulae are applicable.

Please note that $R(t)$ is the same as $a(t)$, the scale factor of the Universe.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>$3.00 \times 10^{10}$ cm/s</td>
</tr>
<tr>
<td>$h$</td>
<td>$6.63 \times 10^{-27}$ erg s</td>
</tr>
<tr>
<td>$G$</td>
<td>$6.67 \times 10^{-8}$ dyne cm$^2$ g$^{-2}$</td>
</tr>
<tr>
<td>$k_B$</td>
<td>$1.38 \times 10^{-16}$ erg/K</td>
</tr>
<tr>
<td>$\sigma_{SB}$</td>
<td>$5.67 \times 10^{-5}$ erg cm$^{-2}$ s$^{-1}$ K$^{-4}$</td>
</tr>
<tr>
<td>$a_{SB}$</td>
<td>$7.56 \times 10^{-15}$ erg cm$^{-3}$ K$^{-4}$</td>
</tr>
<tr>
<td>$m_p$</td>
<td>$1.67 \times 10^{-24}$ g</td>
</tr>
<tr>
<td>$1/H_0$</td>
<td>$9.78 h^{-1}$ Gyr</td>
</tr>
<tr>
<td>$c/H_0$</td>
<td>$3000 h^{-1}$ Mpc</td>
</tr>
<tr>
<td>$1$ pc</td>
<td>$3.09 \times 10^{18}$ cm</td>
</tr>
<tr>
<td>$1$ AU</td>
<td>$1.50 \times 10^{13}$ cm</td>
</tr>
<tr>
<td>$1$ yr</td>
<td>$3.16 \times 10^{7}$ sec</td>
</tr>
<tr>
<td>$1$ M$_\odot$</td>
<td>$1.99 \times 10^{33}$ g</td>
</tr>
<tr>
<td>$1$ $L_{\odot, bol}$</td>
<td>$3.86 \times 10^{33}$ erg/s</td>
</tr>
<tr>
<td>$1$ eV</td>
<td>$1.6 \times 10^{-12}$ erg</td>
</tr>
<tr>
<td>$m_p/m_e$</td>
<td>1836</td>
</tr>
<tr>
<td>$\rho_c$</td>
<td>$1.88 \times 10^{-29}$ g/cm$^3$</td>
</tr>
<tr>
<td>$\rho_c$</td>
<td>$2.78 \times 10^{11}$ M$_\odot/Mpc^3$</td>
</tr>
</tbody>
</table>

$$ds^2 = c^2 dt^2 - R(t)^2 \left[ dr^2 + S(r)^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) \right], \quad \text{where } S(r) = \begin{cases} R_c \sin(r/R_c) & \text{if } r > R_c \\ R_c \sinh(r/R_c) & \text{if } r < R_c \end{cases}$$

$$ds^2 = dt^2 - \frac{R(t)^2}{c^2} \left[ \frac{dr^2}{1 - \kappa r^2} + r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) \right], \quad \text{where } \kappa = \pm R_c^{-2} \text{ or } 0$$

$$H(t) = \frac{1}{R(t)} \frac{dR}{dt}$$

$$r(z) = \int_0^z \frac{c}{H(z)} \, dz$$

$$D_A(z) = \frac{S[r(z)]}{(1 + z)}$$

$$D_L(z) = S[r(z)](1 + z)$$

$$f(E) = \frac{1}{\exp[(E - \mu)/kT] + 1}$$

$$\ddot{R} = - \frac{4\pi G}{3} R \left( \rho + \frac{3p}{c^2} \right) + \frac{1}{3} \Lambda R$$

$$\dot{H} = H_0 \left[ \Omega_{rad}(1 + z)^4 + \Omega_m(1 + z)^3 + \Omega_\Lambda + (1 - \Omega_{rad} - \Omega_m - \Omega_\Lambda)(1 + z)^2 \right]^{1/2}$$

$$\rho c^2 = g_s (a_{SB}/2) T^4$$

$$g_s = (\text{Spin states of relativistic bosons}) + \frac{7}{8} (\text{Spin states of relativistic fermions})$$

$$S[r(z)] = \frac{2c}{H_0 \Omega_0^2 (1 + z)} \left\{ \Omega_0 z + (\Omega_0 - 2) \left[ (\Omega_0 z + 1)^{1/2} - 1 \right] \right\} \quad (\Lambda = 0)$$
\[
\frac{d\rho}{dt} + \rho \nabla_x \cdot \mathbf{v} = 0 \\
\frac{d\mathbf{v}}{dt} = -\frac{1}{\rho} \nabla_x p - \nabla_x \phi \\
\nabla_x \phi = 4\pi G \rho \\
\mathbf{v} = H \mathbf{x} + \delta \mathbf{v} = H \mathbf{x} + R \mathbf{u} \\
\frac{d^2 D}{dt^2} + 2H \frac{dD}{dt} = \left(4\pi G \rho - \frac{c_s^2 k^2}{R^2}\right) D
\]

\[
D(t) \propto 1 + \frac{3}{x} + \frac{3\sqrt{1 + x}}{x^{3/2}} \ln \left[\sqrt{1 + x} - \sqrt{x}\right], \text{where } x = R(t)(1 - \Omega_m)/\Omega_m \text{ and } \Lambda = 0 \\
D(t) \propto (1 + z)^{-1} \text{ for Einstein-de Sitter.}
\]

\[
\sigma^2 = \int \frac{dk}{k} \frac{k^3 P(k)}{2\pi^2} \left|\tilde{W}(k)\right|^2 \text{ where } \tilde{W}(k) = \frac{3kR \cos(kR) - 3 \sin(kR)}{(kR)^3} \text{ for spherical top hat}
\]

\[
\rho_{NFW} \propto \frac{1}{r(r + r_s)^2}, \quad r_{200}/r_s = c \\
\frac{dn}{d\ln M} = \frac{\rho_0}{M} \frac{2}{\pi} \frac{n + 3}{6} |\nu e^{-\nu^2/2}|
\]

\[
\frac{dn}{dL} = \frac{1}{L_s} \left(\frac{L}{L_s}\right)^{\alpha} \exp(-L/L_s)
\]

\[
\Gamma(x) = \int_0^\infty dt \ t^{x-1} e^{-t}
\]

\[
\alpha = \frac{4GM(< b)}{bc^2}
\]

\[
\theta = \frac{d_{LS}}{d_s} \alpha = \beta
\]

\[
\mu = \frac{\theta \frac{d\theta}{d\beta}}{\beta \frac{d\beta}{d\beta}}
\]

\[
s = \int_0^{z_0} c_s(1 + z)dt
\]

\[
\tau_{HI} = A \left(\frac{\rho}{\bar{\rho}}\right)^\beta, \ \beta \approx 1.6
\]

\[
A = 1.13 \left(\frac{1 + z}{4}\right)^6 \left(\frac{\Omega_b h^2}{0.02}\right)^2 \left(\frac{T_0}{2 \times 10^4 K}\right)^{-0.7} \left(\frac{\Gamma_{HI}}{10^{-12} \text{ s}^{-1}}\right)^{-1} \left(\frac{H(z)}{312 \text{ km/s/Mpc}}\right)^{-1}
\]

\[
\frac{d\delta}{dt} + (\mathbf{u} \cdot \nabla_r) \delta + (1 + \delta) \nabla_r \cdot \mathbf{u} = 0 \\
\frac{d\mathbf{u}}{dt} + 2H \mathbf{u} + (\mathbf{u} \cdot \nabla_r) \mathbf{u} = -\frac{c_s^2 \nabla_r \delta}{R^2} + \frac{1}{R^2} \nabla_r \phi \\
\frac{d^2 \phi}{dt^2} = 4\pi G R^2 \rho_0 \delta
\]

\[
\delta \mathbf{v} = \frac{2f}{3H\Omega} \mathbf{g}
\]

\[
f = \frac{a}{D} \frac{dd}{da} \approx \Omega^{0.6}
\]